



Ohio's Learning Standards – Extended with Learning Progressions Mathematics

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Table of Contents

Introduction to Ohio's New Learning Standards - Extended: Mathematics3

OVERVIEW3

 Complexity Levels4

 Learning Progressions.....4

 Accessibility5

Navigating the Ohio Learning Standard Extensions.....6

Navigating the Learning Progressions7

Learning Progressions for Math, Grades K – 28

Kindergarten.....8

Grade 117

Grade 227

Extended Standards with Learning Progressions for Grades 3 - 8.....38

Grade 3 38

Grade 4 52

Grade 5 71

Grade 6 96

Grade 7 121

Grade 8 138

Extended Standards with Learning Progressions for High School 157

Numbers and Quantity..... 157

Algebra 161

Functions 174

Geometry 187

Statistics and Probability 206

Acknowledgements 218

Introduction to Ohio's New Learning Standards - Extended: Mathematics

OVERVIEW

In February 2017, the state of Ohio adopted updated Ohio Learning Standards (OLS) for English language arts and math. Consequently, Ohio revised the Ohio Learning Standards-Extended (OLS-E) to be aligned with the OLS. The Ohio Department of Education collaborated with teams of educators and experts from around the state to do the work. These committees met multiple times to draft the new extensions. The Department then posted the drafts for public feedback and received hundreds of comments. After the public comment period, the committees considered the comments and implemented suggestions into the final version.

The OLS-E are specific statements of knowledge and skills linked to the expectations in the OLS. The purpose of the extensions is to build a bridge that provides grade level access for students with the most significant cognitive disabilities to the content of the OLS.

The Department developed the OLS-E specifically for students who qualify for and take the Alternate Assessment for Students with Significant Cognitive Disabilities (AASCD). These extensions do not replace the OLS for mathematics, they are aligned to them. Teachers may use the standards and extensions as a skill or knowledge progression when designing instruction and assessments. Using a standards progression provides flexible access from varied entry points and allows learners with the most significant cognitive disabilities to grow knowledge and skill across a modified curriculum that is linked to the grade-level standards. Educators can then use the link to

grade-level targets or outcomes as comparison data in present levels of performance on an IEP. Because instruction and assessment should always consider the full range of extended standards and the links to the grade-level targets and outcomes, the OLS-E development committee designed this document so that the reader can reference the OLS and the extensions on the same page to easily see the progression.

While educators should use the extended standards to provide content that is directly aligned to the OLS for mathematics, they must also meet each child's individual education needs by incorporating other skills as necessary. Teachers should consider incorporating instruction with individual accommodations or supports students need to access the curriculum as well as non-academic skills needed for student success such as communication, self-determination, fine/gross motor, and social/emotional skills. Daily living and life skills are often represented within the standards as reading, speaking, listening, writing, and economics skills and should be taught and integrated with the extensions. Educational plans should also include any other additional skills necessary for each child's individual education needs and transition planning goals.

Educators can use the OLS-E to differentiate instruction for a wide range of students by using the extensions as entry points to the OLS, but they must do so with caution. Students who do not take Ohio's AASCD will take the general assessments aligned to the general standards. These extensions can provide entry points into the OLS. However, schools must remember that students who do not participate in the AASCD should transition to and will be assessed using the OLS.

Complexity Levels

The committee extended the Ohio Learning Standards to include three levels from “most complex” to “least complex”. The complexity levels are comprised of three targets of varying difficulty aligned to each standard from the OLS. The extensions are codified individually for clear designation. The last letter in the extension code indicates the complexity level: “a” denotes the highest level of complexity, “b” denotes the middle complexity level and “c” denotes the lowest complexity level. In some instances, the committee tiered the verb of the extension to increase or decrease the complexity level. In other cases, the concept or skill within the OLS is tiered across the three complexity levels. It is important to move from left to right when reading the extensions. To determine where instruction should begin, educators should start with the general standard and then progress down through the complexity levels until finding the optimum starting point. It’s important to note that no one should categorize students according to an extension level. Instead, instruction should build skills across the extensions to the highest level possible based on individual student strengths which may vary across standards. Ideally, when educators apply these extensions within each grade level one should see instruction occurring at all ranges of complexity. When citing standards for lesson and/or assessment design, educators should include the full complexity range, including the general standard. Citing standards in this way acknowledges a range of entry points and a range of learning progressions

*Additional mathematics standards for High School that represent complex numbers in the Ohio’s New Learning Standards are not included in the extended standards since they are not considered common mathematics curriculum for all college and career ready students.

Learning Progressions

A learning progression is a sequence of skills linked to a learning target that build base skills and engagement as learners make progress toward mastery of the standard or learning target.

These Learning Progressions are a companion to Ohio’s New Learning Standards and OLS-E and help develop teacher and learner clarity about embedded skills within each standard. Learning progressions are building blocks that can outline how learning builds before, over the course of, and after the target skill.

*It is worth noting that none of the above can be crafted unless the educator has first identified the learning target or standard of focus for the lesson and assessment.

Ohio’s Learning Progressions companion document includes the Kindergarten, First and Second grade standards that are not part of Ohio’s Learning Standards- Extended. As a reminder, there are no extended standards for grades K-2, however, the learning progressions outline essential skills that are part of each standard.

Together the general standard, extended standards, building base skills and engagement statements make up these sample learning progressions. Learning progressions can be used by education professionals, learners and their families to identify entry level skills, set goals and objectives and track progress over time. Please note: Skills provided in each progression are a sampling of skills leading toward mastery and are not intended to be an all inclusive list. It is possible that other skills may be part of each learning progression and may not all be included in this document.

Skills in the learning progression column are organized in a learning ladder, beginning at the bottom with engagement indicators and moving upward in complexity toward mastery. Some learning progressions may include skills that fall between extended standard complexity levels in addition to building base

skills leading to Complexity C. Learning Progressions in grades K-2 include engagement indicators, base skills and a deconstruction of the grade level target.

Learning progressions can be helpful in designing assessments, measuring progress over the course of learning and identifying baseline skills and knowledge. In addition, when used as a data collection tool paired with evidence of student learning, the learning progression may reveal skill gaps and intervention targets. Large segments of missing skills may be considered by intervention and IEP teams as potential goals and objectives that may be targeted to close achievement gaps between the learner and grade-level peers. Thus, leading to individualized IEP development aligned with standards-based data. Data aligned to learning progressions can also be used to articulate present levels by citing skills that the learner has demonstrated leading up to a skill gap, also known as present levels of performance.

Learning progressions help education professionals maintain age/grade-level alignment for each learner while still addressing both strengths and needs. No matter where the entry point, all learners can demonstrate skills leading to grade-level outcomes. Seeing each and every learner as part of the learning continuum at grade-level is the goal. Learning progressions help make that goal a reality for all learners and all teaching professionals.

Please note: This resource provides many, but not all skills leading to and through Ohio's Learning Standards and OLS-E. The OLS-E with Learning Progressions is a living document. Review and refinement of these LP's is ongoing and the documents will be updated regularly. Please check in frequently for the most up to date version.

Accessibility

The OLS-E and the Learning Progressions do not specify individual accommodations or supports that may be necessary for students to access the curriculum. Teachers should consider the unique learning needs of each student and integrate the Individualized Education Plan (IEP) designated supports and services when designing lessons. It is imperative that teachers provide specially designed instruction, assistive technology, accommodations and other supports needed to ensure full access to learning opportunities so that students can demonstrate their full range of knowledge and skills.

Navigating the Ohio Learning Standard Extensions

The graphic illustrates the components of the Extensions and Learning Progressions:

GRADE 3

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex			Least Complex
Operations and Algebraic Thinking			
<i>Represent and solve problems involving multiplication and division.</i>			
3.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. (Note: These standards are written with the convention that $a \times b$ means a groups of b objects each; however, because of the commutative property, students may also interpret 5×7 as the total number of objects in 7 groups of 5 objects each.)	3.OA.1a Represent products of whole numbers up to 10×10 using arrays, area models, or physical objects (whole numbers 0 through 10).	3.OA.1b Represent products with factors of 1s, 2s, 3s, 4s, 5s, and 10s using arrays, area models, or physical objects (whole numbers 0 through 10).	3.OA.1c Represent products with factors of 1s, 2s, and 5s using arrays, area models, or physical objects (whole numbers 0 through 10).
3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of shares when 56 objects are partitioned equally into 8 shares, or as the number of groups of 8 objects each. For example, $56 \div 8 = 7$ means that 56 can be expressed as 8 groups of 7.	3.OA.2a Represent quotients of single-digit whole numbers up to 100 (including 0).	3.OA.2b Represent quotients using arrays, area models, or other physical representations for whole number factors of 1s, 2s, 3s, 4s, 5s, and 10s with products not exceeding 100.	3.OA.2c Represent quotients using arrays, area models, or other physical representations for whole numbers factors of 1s, 2s, and 5s with products not exceeding 10, 20, and 50, respectively.

Three levels of complexity

Grade

Domain

Cluster

Standards with Codification

Navigating the Learning Progressions

The graphic illustrates the components of the Learning Progression:

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
				Least Complex
<p>3.LS.2.a1 Describe how a variation in a trait would give an individual an advantage.</p> <p>3.LS.2.a2 Explain why one physical or behavioral trait within a species would be an advantage in a given environment.</p>	<p>3.LS.2.a1 Describe how a variation in a trait would give an individual an advantage.</p> <p>3.LS.2.a2 Explain why one physical or behavioral trait within a species would be an advantage in a given environment.</p>	<p>3.LS.2.b1 Identify variations in physical and behavioral traits within a species.</p> <p>3.LS.2.b2 When given an environment and given variations of the same organism, select the one variation that will be an advantage to the organism.</p>	<p>3.LS.2.c1 Identify that there is variation within the population.</p> <p>3.LS.2.c2 Identify traits in an individual organism that would help it be successful in its environment.</p>	<ul style="list-style-type: none"> Explain why a particular trait is advantageous in a given environment. Identify traits as advantageous in certain environments. Describe the traits of an organism that help it live in its environment (e.g., thick covering on cactus, the ability to hibernate in Arctic animals, the ability to dig, fast running speed of prey, ability to climb). Identify that variation may provide a survival advantage. Match organisms to an environment in which they can survive.
<p>3.LS.3 Plan and conduct an investigation to provide evidence of the life cycle of an organism, which includes birth, growth, adulthood, reproduction, and death.</p> <p>3.LS.3.a Plan and conduct an investigation to provide evidence of the life cycle of an organism, which includes birth, growth, adulthood, reproduction, and death.</p>	<p>3.LS.3.a1 Plan and conduct an investigation to provide evidence of the life cycle of an organism, which includes birth, growth, adulthood, reproduction, and death.</p>	<p>3.LS.3.b1 Match an organism to its life cycle.</p>	<p>3.LS.3.c Identify the main stages of an organism's life cycle: birth, growth, adulthood, reproduction, and death.</p>	<ul style="list-style-type: none"> Recognize that different organisms may have different life stages (e.g., metamorphosis). Match stage of the life cycle to what is going on in the environment (e.g., many babies are born in the spring when food is plentiful, seeds germinate when the soil is moist). Know that an individual organism may die at any point in the life cycle. Identify different stages of a life cycle (e.g., watch nature videos that trace the life cycle of an organism).

Building base skills

This learning progression includes the general standard, 3 extended standards and base skills beginning with engagement

Skills grow from engagement and base skills (at the bottom) to and sometimes through the extended standards (moving upward like a learning ladder)

Is important to note that each standard and extended standard are made up of many individual points of learning (knowledge and skills). All of these skills can be developed and monitored with explicit instruction and assessment.

Learning Progressions for Math, Grades K – 2

Kindergarten

Grade-Level Standard Kindergarten	Learning Progression <i>Building the Base & Engagement</i>
Counting and Cardinality	
<p>Know number names and the count sequence. K.CC.1 Count to 100 by ones and by tens.</p>	<ul style="list-style-type: none"> Count by 1s up to 75 Count by 10s to 100 using a model to represent groups of 10 Count by 1s up to 50 Count by 10s to 50 using a model to represent groups of 10 Count up to 10 by 1s using a model or concrete objects with 1:1 correspondence Match the number 10 to a model to represent group of 10 (base 10 block, ten frame, etc.) Count by 1s to 10 Engage with numbers by 10 (10, 20, 30, 40, etc.) Engage with numbers 1-10.
<p>K.CC.2 Count forward within 100 beginning from any given number other than 1.</p>	<ul style="list-style-type: none"> Count forward 10 more beginning with any multiple of 10 up to 90 Count forward beginning from a given number between 20 and 50 Count forward 5 more given a starting number from 2-20 Identify the next number when given a starting number between 1 and 10 Order a set of given numbers in sequence beginning with any number other than one Match numbers that are side-by side in a sequence Engage with numbers that are side-by side in a counting sequence Engage with a number line or 100 chart
<p>K.CC.3 Write numerals from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).</p>	<ul style="list-style-type: none"> Write/identify numbers 11-20 in random order when given a number name. Write/identify numbers 11-20 in consecutive order. Write/identify numbers 0-10 in random order when given a number name. Write/identify numbers 0-10 in consecutive order. Write/identify the numbers (within a range of 0-20) to represent a number of objects. Write/identify a number (within a range of 0-10) to represent a number of objects. Write/identify each number while counting objects with 1:1 correspondence. Match the correct numeral for a set of objects up to 20, including 0. Match written number to quantity of objects up to 20. Match written number to quantity of objects up to 10.

Grade-Level Standard Kindergarten	Learning Progression Building the Base & Engagement
	<ul style="list-style-type: none"> • Match a spoken number to quantity of objects up to 20. • Match a spoken number to quantity of objects up to 10. • Identify numbers with 1:1 correspondence while orally counting up to 20. • Identify numbers with 1:1 correspondence while orally counting up to 10. • Identify 0 as representing no objects. • Engage with tools for writing or communicating numbers. • Engage with numbers (written or tactile).
<p>Count to tell the number of objects. K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality using a variety of objects including pennies. a. When counting objects, establish a one-to-one relationship by saying the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted and that the number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.</p>	<ul style="list-style-type: none"> • Describe/identify that order/placement of the same group of objects does not change the sum of the group. • Count objects in a given group from different starting places and compare the sum each time with the previous count. • Visually/tactily compare multiple arrangements of objects with the same sum. • State/identify the last number counted as the sum of a group of objects. • Identify the last number counted in a group of objects. • Identify which number in a sequential set is more. • Demonstrate more using objects and numbers in a sequence (show that 4 is more than 3, or 8 is more than 7) • Describe/identify that each number named in the sequence indicates “more” of a given object. • Identify and say each number while counting objects with 1:1 correspondence. • Using a visual or tactile organizer, match a line of objects with its corresponding number in a counting sequence. • Identify numbers 11-20 by name. • Match the correct numeral for objects up to 20, including 0. • Count the total number of objects up to 20. • Match the correct numeral to objects up to 15. • Identify numbers 0-10 by name. • Match the correct numeral to objects up to 10. • Count the total number of objects up to 10. • Match the correct numeral to objects up to 5. • Count the total number of objects up to 5. • Engage with numbers 0-20 (written or tactile). • Engage with a group of objects representing a numbered quantity.
<p>K.CC.5 Count to answer “how many?” questions about as many as 20 things</p>	<ul style="list-style-type: none"> • Answer the question of “How many” objects are in this group? • State/identify the last number counted as the sum of a group of objects. • Identify the last number counted in a group of objects.

Grade-Level Standard Kindergarten	Learning Progression Building the Base & Engagement
<p>arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.</p>	<ul style="list-style-type: none"> Count objects in a random group. Count objects in a shaped (square, rectangle, circle) group. Count objects in a linear group. Given a number from 11-20 count out that many objects. Given a number from 6-10 count out that many objects. Given a number from 1-5 count out that many objects. Identify that numbers of objects can be grouped in different ways (i.e. rows or lines, rectangles or squares, circles and/or random scattered patterns). Identify that a question containing "how many" refers to numbers and counting. Engage with numbers 0-20 (auditory or tactile). Engage in counting activities.
<p>Compare numbers. K.CC.6 Orally identify (without using inequality symbols) whether the number of objects in one group is greater/more than, less/fewer than, or the same as the number of objects in another group, not to exceed 10 objects in each group.</p>	<ul style="list-style-type: none"> Identify whether the number of objects in one group is "greater than," "less than," or the same as the objects in another group, for up to 10 objects. Compare and describe two collections of objects presented in the same visual/tactile organizers using terms such as more, less, or same. Count objects presented in two groups using the same visual/tactile organizers and record the number in each group. Using common visual/tactile organizers, place objects from each of the two collections into individual organizers. Engage in comparing two collections of objects. Engage in counting activities. Engage with collections of objects not to exceed 10.
<p>K.CC.7 Compare (without using inequality symbols) two numbers between 0 and 10 when presented as written numerals.</p>	<ul style="list-style-type: none"> Compare two numerals between 1 and 10 to determine which is "greater than", "less than", or "equal to." Compare two numerals between 1 and 10 to determine which is "greater than" or "less than." Compare and describe two groups of objects presented in the same visual/tactile organizers using terms such as more, less, or same. Using visual/tactile models, show two numbers presented side by side. Match a given number from 0-10 with a visual/tactile model. Match comparison vocabulary such as more, less and same to object or picture models showing groups of 0-10. Identify comparison vocabulary as more, less and same. Engage in comparing two numbers. Engage with numbers 1-10 (auditory or tactile). Engage in counting activities not to exceed 10.

Grade-Level Standard Kindergarten	Learning Progression <i>Building the Base & Engagement</i>
Operations and Algebraic Thinking	
<p>Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.</p> <p>K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds such as claps, acting out situations, verbal explanations, expressions, or equations. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)</p>	<ul style="list-style-type: none"> • Represent addition and subtraction using auditory, visual and/or tactile models (i.e. fingers, tally marks, touchpoints, objects, counters, tapping, chants/rhymes/songs, etc.) • Match subtract (-) to terms such as: take away, take apart, minus, subtract • Match add (+) to terms such as: count on, plus, put together, group. • Describe or demonstrate - as taking away using multi-sensory models. • Describe or demonstrate + as putting together using multi-sensory models. • Follow along and mimic “putting together” and “taking apart” as demonstrated using multi-sensory models to represent addition and subtraction. • Engage in demonstrations using multi-sensory models to represent addition and subtraction. • Engage in auditory, visual and/or tactile models (i.e. fingers, tally marks, touchpoints, objects, counters, tapping, chants/rhymes/songs, etc.) representing addition and subtraction.
<p>K.OA.2 Solve addition and subtraction problems (written or oral), and add and subtract within 10 by using objects or drawings to represent the problem.</p>	<ul style="list-style-type: none"> • Solve addition and subtraction problems within 10 involving situations where one is “adding to,” “taking from,” “putting together” and “taking apart,” using models or objects. • Solve addition and subtraction problems within 5, involving situations where one is “adding to,” “taking from,” “putting together” and “taking apart,” using models or objects. • Demonstrate addition as “putting together” and subtraction as “taking away,” using models or objects up to a sum of 10. • Use a number sentence to subtract/take away one number from another and count the sum of 10 or less. • Match a number sentence to a like object model representing addition and subtraction problems with a sum of 10 or less. • Identify a number sentence involving subtraction up to 10. (i.e. $3-1=$ or $9+1=$) • Use object model to subtract/take away one group of objects from whole and count the remaining objects with a sum of 10 or less. • Use a number sentence to add/put together two numbers to count to a sum of 10 or less. • Match a number sentence to a like object model involving problems with a sum of 10 or less. • Identify a number sentence involving addition. ($4+1=$) • Use object model to add/put together two groups and count the sum. • Identify numbers 1-5. • Describe or demonstrate - as taking away. • Describe or demonstrate + as putting together. • Name the + and - signs. (can use terms such as: add, plus, take away, minus, subtract)

Grade-Level Standard Kindergarten	Learning Progression <i>Building the Base & Engagement</i>
	<ul style="list-style-type: none"> Engage with the signs for + and -. Engage with numbers.
K.OA.3 Decompose numbers and record compositions for numbers less than or equal to 10 into pairs in more than one way by using objects and, when appropriate, drawings or equations.	<ul style="list-style-type: none"> Use number labels from a visual/tactile models of the decomposed number ten to record the composition or number sentence (i.e. $5+5=10$) Using a visual/tactile model of the decomposed number ten into any two groups label each group with its number representation Decompose a visual/tactile model of the number ten into any two groups (i.e. 1 and 9, 4 and 6, 3 and 7, etc.) Create a visual/tactile model of the number ten Identify numbers 1-9 Select a visual/tactile model (ten frame, unifix cubes, base ten rod, etc.) representing 10 Engage with the numbers 1-9 Engage with the number 10 Engage with a model representing the number 10 (ten frame, base 10 block, etc.)
K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or, when appropriate, an equation.	<ul style="list-style-type: none"> Select or write an equation to represent the sum of ten ($6+4=10$) Use color change, object change or other change feature to count up to 10 from a given number 1-9 <div data-bbox="619 846 1220 980"> </div> <ul style="list-style-type: none"> Identify numbers 1-9 as less than ten Use a visual model to compare numbers 1-9 with a model of 10 Identify numbers 1-9 Select a visual/tactile model (ten frame, unifix cubes, base ten rod, etc.) representing 10 Engage with the numbers 1-9 Engage with the number 10 Engage with a model representing the number 10 (ten frame, base 10 blocks, etc.)
K.OA.5 Fluently add and subtract within 5.	<ul style="list-style-type: none"> Use a number sentence to subtract/take away one number from another and count the sum. Match a number sentence to a like object model. Identify a number sentence involving subtraction. ($3-1=$) Use object model to subtract/take away one group of objects from whole and count the remaining objects. Use a number sentence to add/put together two numbers to count the sum. Match a number sentence to a like object model.

Grade-Level Standard <i>Kindergarten</i>	Learning Progression <i>Building the Base & Engagement</i>
	<ul style="list-style-type: none"> • Identify a number sentence involving addition. ($4+1=$) • Use object model to add/put together two groups and count the sum. • Identify numbers 1-5. • Describe or demonstrate - as taking away. • Describe or demonstrate + as putting together. • Name the + and - signs. (can use terms such as: add, plus, take away, minus, subtract) • Engage with the signs for + and -. • Engage with numbers.
<i>Number and Operations Base Ten</i>	
<p>Work with numbers 11–19 to gain foundations for place value. K.NBT.1 Compose and decompose numbers from 11 to 19 into a group of ten ones and some further ones by using objects and, when appropriate, drawings or equations; understand that these numbers are composed of a group of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.</p>	<ul style="list-style-type: none"> • Decompose a number from 11-19 into ten and ones. (i.e. $11 = 10+1$ or one 10 and one, one) • Use ten frames to compose and decompose a number up to 19. • Decompose an object model into a group of ten and ones up to 19. • Compose an object model for numbers 0-19. • Use 1:1 correspondence to count objects, or models of objects, representing a two-digit number up to 19. • Use 1:1 correspondence to count objects, or models of objects, representing a two-digit number up to 10. • Identify given numbers up to 19. • Count by 1s to 19. • Count by 5's to 10 • Count by 1's to 10 • Participate in counting activities. • Engage with number models. • Engage with objects to be counted.
<i>Measurement and Data</i>	
<p>Identify, describe, and compare measurable attributes. K.MD.1 Identify and describe measurable attributes (length, weight, and height) of a single object using vocabulary terms such as long/short, heavy/light, or tall/short.</p>	<ul style="list-style-type: none"> • Use measurement vocabulary to describe an object. • Identify opposite attributes (heavy/light, tall/short, long/short, etc.) • Identify weight, height, length, size, etc. as measurable attributes. • Match the terms long/short/tall with length. • Match the terms heavy/light with weight. • Engage in conversations about objects. • Engage with an object that can be measured by weight or length.

Grade-Level Standard <i>Kindergarten</i>	Learning Progression <i>Building the Base & Engagement</i>
<p>K.MD.2 Directly compare two objects with a measurable attribute in common to see which object has “more of” or “less of” the attribute, and describe the difference. <i>For example, directly compare the heights of two children, and describe one child as taller/shorter.</i></p>	<ul style="list-style-type: none"> • Compare two objects and identify as longer, shorter, taller or same length. • Identify weight, height, length, size, etc. as measurable attributes. • Place two objects side-by-side. • Actively engage with two objects • Actively engage with one object
<p>Classify objects and count the number of objects in each category. K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. The number of objects in each category should be less than or equal to ten. Counting and sorting coins should be limited to pennies.</p>	<ul style="list-style-type: none"> • Match two groups of objects with the same count. • Count group of objects between 6-10. • Identify group of objects as 6, 7, 8, 9, or 10. • Count group of objects between 1-5. • Identify group of objects as 0, 1, 2, 3, 4 or 5. • Count objects presented in a ten frame. • Following a model, count objects chorally using 1:1 correspondence, giving the number in association with the object. • Orally count, sign or count using assistive technology, with 1 second automaticity from 1-10 • Participate in counting activities. (using assistive technology switches or voice output devices as needed) • Engage in counting activities, songs, rhymes, tools. (numbers 1-10)
<i>Geometry</i>	
<p>Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres). K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as <i>above, below, beside, in front of, behind, and next to.</i></p>	<ul style="list-style-type: none"> • Describe the relative positions of objects using terms such as “above,” “below,” “beside,” “in front,” “behind” and “next to.” • Sort objects in the environment by their shape. • Match objects in the environment to a given shape. • Match a shape from a given array to an object from the environment. • Describe an object as having shape. • Using the terms above, below, beside, in front of, behind, and next to to describe where the object was located in the environment. • Name an object from the environment. • Select an object from the environment to describe. • Engage with objects from the environment.

Grade-Level Standard Kindergarten	Learning Progression Building the Base & Engagement
K.G.2 Correctly name shapes regardless of their orientations or overall size.	<ul style="list-style-type: none"> • Match shapes of differing orientation to the name of the shape. • Match shapes of all sizes to the name of the shape. • Name the shapes squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres. • Identify squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres given the name of the shape. • Identify squares, triangles, circles and rectangles given the name of the shape. • Explore and manipulate shapes tactually or visually. • Engage in activities involving shapes.
K.G.3 Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).	<ul style="list-style-type: none"> • Identify shapes as two-dimensional or three- dimensional (i.e., flat vs. solid). • Sort two dimensional and three-dimensional shapes/objects. • Sort/match pictures as flat and objects as solid. • Learn vocabulary associated with dimension: flat, solid. • Engage with two or three dimensional shapes.
Describe, compare, create, and compose shapes. K.G.4 Describe and compare two- or three-dimensional shapes, in different sizes and orientations, using informal language to describe their commonalities, differences, parts, and other attributes.	<ul style="list-style-type: none"> • Describe two or three dimensional objects using vocabulary to tell about size, shape, parts, similarities or differences. • Group given shapes by common attributes (e.g. number of sides, 2D or 3D, curved lines, number of corners (angles), parallel lines, right angles, etc.) • Identify the characteristics of a three dimensional shape • Identify the characteristics of a two dimensional shape • Name parts of a given shape (e.g. base, side, cone, line, corner, etc.) • Identify parts of shapes given the name (e.g. base, side, cone, line, corner, etc.) • Order given shapes by size • Learn vocabulary associated with size such as small, big, medium, large, extra large, largest, smallest, etc. • Name shapes shown in different orientations • Manipulate a 2D or 3D shapes by turning or repositioning • Engage with two or three dimensional objects
K.G.5 Model shapes in the world by building shapes from components, e.g., sticks and clay balls, and drawing shapes.	<ul style="list-style-type: none"> • Build a shape seen in the world using given materials • Match simple shapes to pictures of objects from the environment • Identify shapes seen in the world through given pictures or other experiences • Follow step-by-step directions to use building materials to create a shape • Identify building verbs (e.g. build, roll, draw, etc.) • Identify building materials by name (e.g. sticks, rod, clay, ball, pencil, etc.) • Participate in building activities

Grade-Level Standard Kindergarten	Learning Progression <i>Building the Base & Engagement</i>
	<ul style="list-style-type: none"> • Engage in building activities • Engage with building materials
K.G.6 Combine simple shapes to form larger shapes.	<ul style="list-style-type: none"> • Compose simple shapes from other basic shapes (e.g., a rectangle can be composed from two right triangles). • Match shapes that can be combined to form new shapes (e.g. triangles can be used to form diamonds or squares, squares can be used to make rectangles, but not circles) • Name simple shapes given the visually or tactually • Identify simple shapes (e.g. circle, square, triangle, diamond, rectangle) when given the name • Identify a shape vs. letter or number • Engage in activities involving shapes

Grade 1

Grade-Level Standard Grade 1	Learning Progression <i>Building the Base & Engagement</i>
<i>Operations and Algebraic Thinking</i>	
<p>Represent and solve problems involving addition and subtraction. 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. See Table 1, page 95.</p>	<ul style="list-style-type: none"> Solve subtraction word problems within 20, involving situations where one is “taking from,” or “taking apart,” using models or objects. Solve addition problems within 20, involving situations where one is “adding to,” or “putting together” using models or objects. Select and use numbers and operation symbols presented in a given word problem to generate a simple equation with a sum of 20 or less. Solve subtraction word problems within 10, involving situations where one is “taking from,” or “taking apart,” using models or objects. Solve addition word problems within 10, involving situations where one is “adding to,” or “putting together” using models or objects. Select and use numbers and operation symbols presented in a given word problem to generate a simple equation with a sum of 10 or less. Manipulate a model or objects, as described in a story problem, to identify the unknown part of the equation. Using an equation template, identify the part of the equation that is unknown as described in a story problem. Using an equation template, identify the operation (+ or -) as described in a story problem. Using an equation template, identify the given numbers as described in a story problem. Identify within a passage of text the given numbers, the operation and the problem to solve (the unknown). Understand that a passage of text can contain a math problem. Demonstrate addition and subtraction using visual and/or tactile models and simple equations (i.e. drawings, tally marks, touchpoints, objects, counters, etc.) Match subtract (-) to terms such as: take away, take apart, minus, subtract Match add (+) to terms such as: count on, plus, put together, group. Describe or demonstrate - as taking away using multi-sensory models. Describe or demonstrate + as putting together using multi-sensory models. Follow along and mimic “putting together” and “taking apart” as demonstrated using multi-sensory models to represent addition and subtraction. Engage in demonstrations using multi-sensory models to represent addition and subtraction equations. Engage in visual and/or tactile models (i.e. drawings, tally marks, touchpoints, objects, counters, etc.) representing addition and subtraction equations.
<p>1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20,</p>	<ul style="list-style-type: none"> Independently add three whole numbers given in a word problem (written or oral), up to a sum of 20 or less, with or without models or objects. Add three whole numbers to a sum of 20 or less.

Grade-Level Standard Grade 1	Learning Progression Building the Base & Engagement
<p>e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)</p>	<ul style="list-style-type: none"> Solve addition problems within 20, involving situations where one is “adding to,” or “putting together” using models or objects. Select and use numbers and operation symbols presented in a given word problem to generate an addition equation using 3 whole numbers with a sum of 20 or less. Solve addition word problems within 10, involving situations where one is “adding to,” or “putting together” using models or objects. Select and use numbers and operation symbols presented in a given word problem to generate an addition equation using 3 whole numbers with a sum of 10 or less. Manipulate a model or objects, as described in a story problem, to identify the unknown part of the equation. Using an equation template, identify the part of the equation that is unknown as described in a story problem. Using an equation template, identify the operation (+) as described in a story problem. Using an equation template, identify the given numbers as described in a story problem. Identify within a passage of text the given numbers, the operation and the problem to solve (the unknown). Understand that a passage of text can contain a math problem. Demonstrate addition using visual and/or tactile models and simple equations (i.e. drawings, tally marks, touchpoints, objects, counters, etc.) Match add (+) to terms such as: count on, plus, put together, group. Describe or demonstrate + as putting together using multi-sensory models. Follow along and mimic “putting together” as demonstrated using multi-sensory models to represent addition problems. Engage in demonstrations using multi-sensory models to represent addition and subtraction equations. Engage in visual and/or tactile models (i.e. drawings, tally marks, touchpoints, objects, counters, etc.) representing addition and subtraction equations.
<p>Understand and apply properties of operations and the relationship between addition and subtraction. 1.OA.3 Apply properties of operations as strategies to add and subtract. <i>For example, if $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known (Commutative Property of Addition); to add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 =$</i></p>	<ul style="list-style-type: none"> Apply knowledge of addition facts to solve addition problems involving 3 single digit numbers. (i.e. $5+3+5=?$ by adding $5+5=10$ as a known fact, the problem then becomes $10+3=?$) When given a 3 number addition problem, rewrite the problem as a two number problem by including the answer to the known addition fact. (i.e. $2+4+8=$ becomes $6+8=$) When given a 3 number addition problem, identify two numbers that when added are a know fact. Solve inverse pairs of math facts with increased increments of automaticity from an individual baseline time. Match fact pairs showing inverse subtraction relationships. (i.e. $7-3=4$ and $7-4=3$) Match fact pairs showing inverse addition relationships. Engage with pairs of reversed addition facts (i.e. $2+3=5$ and $3+2=5$). Engage with numbers presented in equation format.

Grade-Level Standard Grade 1	Learning Progression Building the Base & Engagement
<p>$2 + 10 = 12$ (Associative Property of Addition). Students need not use formal terms for these properties.</p>	
<p>1.OA.4 Understand subtraction as an unknown-addend problem. <i>For example, subtract $10 - 8$ by finding the number that makes 10 when added to 8.</i></p>	<ul style="list-style-type: none"> • Using the relationship between addends, identify the missing number in two facts. (i.e. if $4+5=9$ then $9-?=5$) • Identify that a subtraction problem such as $10-8=2$ and the addition problem such as $8+2=10$ are related by the same set of numbers. • Identify = symbol. • Identify + symbol. • Identify subtraction symbol. • Engage with subtraction facts. • Engage with numbers presented in equations.
<p>Add and subtract within 20. 1.OA.5 Relate counting to addition and subtraction, e.g., by counting on 2 to add 2.</p>	<ul style="list-style-type: none"> • Independently count backward to subtract a number from a given starting place on a number line. • Independently count on to add a number to a given starting place on a number line. • Count backward, using a visual model, from a given number (20 or lower) to represent subtraction. • Use a number line to count backward from 20-0. • Use a number line to count backward from 10-0. • Demonstrate/identify which way to move on the number line to subtract (count backward). • Count on from a given number to represent addition up to 20. • Demonstrate/identify which way to move on the number line to add (count on). • Match subtract (-) to the terms count backward. • Match addition (+) to the terms count on. • Count from memory from 0-20. • Count from memory from 0-10. • Use a number line to count from 0-5, 0-10, 0-15, 0--20. • Identify numbers on a number line. • Engage with numbers presented on a number line.
<p>1.OA.6 Add and subtract within 20, demonstrating fluency⁶ with various strategies for addition and subtraction within 10. Strategies may include counting on; making ten, e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$; decomposing a number leading to a ten, e.g., $13 - 4 =$</p>	<ul style="list-style-type: none"> • Solve subtraction math facts within 20 with increased increments of automaticity from an individual baseline time. • Solve addition math facts within 20 with increased increments of automaticity from an individual baseline time. <p>AND/OR</p> <ul style="list-style-type: none"> • Use the relationship between addition and subtraction to solve math facts with increased increments of automaticity from an individual baseline time. (i.e. if $4+3=7$ then $7-3=4$)

Grade-Level Standard Grade 1	Learning Progression Building the Base & Engagement
<p>13 – 3 – 1 = 10 – 1 = 9; using the relationship between addition and subtraction, e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4; and creating equivalent but easier or known sums, e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13.</p>	<ul style="list-style-type: none"> • Use inverse pairs of math facts to increased increments of automaticity from an individual baseline time. (i.e. use 4+5=9 to answer 5+4=?) • Use counting on and counting backward to increased increments of automaticity from an individual baseline time. • Use decomposing numbers to solve addition and subtraction problems with increased increments of automaticity from an individual baseline time (i.e. 18-5=? Use: (10+5+3)-5= 13). • Use known sums to solve addition and subtraction problems with increased increments of automaticity from an individual baseline time.(i.e. If 5+5 is known then use 5+5+2 to solve 5+7=) • Identify = symbol. • Identify + symbol. • Identify subtraction symbol. • Engage with subtraction facts. • Engage with numbers presented in equations.
<p>Work with addition and subtraction equations. 1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. <i>For example, which of the following equations are true and which are false? 6 = 6; 7 = 8 – 1; 5 + 2 = 2 + 5; 4 + 1 = 5 + 2.</i></p>	<ul style="list-style-type: none"> • Determine whether equations involving addition and subtraction within 20 are true or false (e.g., 2 + 3 = 4 + 1). • Solve two simple equations on either side of an equal sign to determine if they are or are not equal. (e.g., 2 + 3 = 4 + 1 is 5=5 and is equal). • Determine whether an equality statement is true or false using objects and/or numerals. • Determine whether one set of numbers is equal to another set of numbers. • Determine whether one set of objects is equal to another set of objects. • Identify equal sign to mean the same or equal to. • Identify addition sign to mean putting together. • Identify subtraction sign to mean take away. • Identify an equation as a numbers sentence containing symbols, including the = sign, that represents a mathematical problem. • Engage with activities involving comparisons. • Engage with visual or tactile representations of the = sign. • Engage with numbers presented in equations.
<p>1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. <i>For example, determine the unknown number that makes the equation true in each of the equations: 8 + = 11; 5 = – 3; 6 + 6 =.</i></p>	<ul style="list-style-type: none"> • Determine the unknown number that makes a subtraction equation true up to a sum of 20. • Determine the unknown number that makes an addition equation true up to a sum of 20. • Determine the unknown number that makes a subtraction equation true up to a sum of 10. • Determine the unknown number that makes an addition equation true up to a sum of 10. • Determine the unknown sum that makes a subtraction equation true. • Determine the unknown sum that makes an addition equation true.

Grade-Level Standard Grade 1	Learning Progression <i>Building the Base & Engagement</i>
	<ul style="list-style-type: none"> Identify/locate the space representing the unknown number in a given equation. Identify that unknown numbers can be in different types of equations (+ or -) Identify that the unknown number can be located in several areas within an equation (i.e. $2+3=?$ or $?+3=5$ or $2+?=5$) Identify the blank line, question mark or open box as a placeholder for an unknown number. Engage with equations containing numbers and blank spaces for unknown numbers. Engage with whole numbers.
<i>Number and Operations Base Ten</i>	
<p>Extend the counting sequence. 1.NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.</p>	<ul style="list-style-type: none"> Write/identify numbers up to 120. Count on from any number to 120. Count by 10's to 120 starting at any multiple of ten. Use visual/tactile models representing groups of ten to count up to 120. Identify models that represent the number 10. (i.e. ten frame, base ten rod, tally marks, etc.) Match a number of objects with a written number. Use 1:1 correspondence to count objects, or models of objects, representing a two-digit number up to 50. Identify/read given numbers up to 120. Count by 1's to 100. Participate in counting activities. Engage with counting tools. Engage with objects to be counted.
<p>Understand place value. 1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones — called a “ten;” the numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones; and the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).</p>	<ul style="list-style-type: none"> Identify multiples of ten as being two, three, four, etc. groups of ten. Use ten frames to represent groups containing ten ones that can be counted by tens. Compose a model representing a two digit number using groups of ten and ones. Compose a model representing a one digit number using single objects or models. Identify models that represent groups of 10. (i.e. ten frame, base ten rod, tally marks, etc.) Identify that two-digit numbers contain groups of tens and ones. Engage with two-digit numbers. Engage with number models representing ten. Engage with the number ten. Engage with numbers 1-9.

Grade-Level Standard Grade 1	Learning Progression Building the Base & Engagement
<p>1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols $>$, $=$, and $<$.</p>	<ul style="list-style-type: none"> • Compare 2 two-digit numbers using the symbols $>$, $=$, and $<$. • Compare quantities within two groups of objects up to 10 using the vocabulary “more than,” “less than” or the “same as.” • Compare two objects and identify as longer, shorter, taller or same length. • Count two groups of objects and label each group with the sum. • Place two groups of objects side-by-side. • Match the symbol $<$ with less than. • Match the symbol $>$ with greater than or more than. • Match the symbol $=$ with same as or equal. • Actively engage with the symbols $>$, $=$, and $<$. • Actively engage with two digit numbers or objects representing two digit numbers.
<p>Use place value understanding and properties of operations to add and subtract.</p> <p>1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; record the strategy with a written numerical method (drawings and, when appropriate, equations) and explain the reasoning used. Understand that when adding two-digit numbers, tens are added to tens; ones are added to ones; and sometimes it is necessary to compose a ten.</p>	<ul style="list-style-type: none"> • Describe why a specific strategy was selected to add 2 two-digit numbers. • Identify when a strategy is used to add 2 two-digit numbers and name the strategy selected. • Use a strategies such as place value, relationship between addition and subtraction, use of decomposing numbers into tens, etc. to add 2 two-digit numbers with a sum of 99 or less. • Use a strategies such as place value, relationship between addition and subtraction, use of decomposing numbers into tens, etc. to add a two-digit number with a one-digit number to a sum of 99 or less. • Use a visual/tactile model to add/put together a two-digit number with a multiple of ten to a sum of 99 or less. • Describe/outline the order of operations used when adding 2 two-digit numbers. (add the ones to the ones then write the sum below the ones column and then add the tens to the tens and write that sum below the tens column. • Write/organize an addition problem using place value alignment. (i.e. $\begin{array}{r} 48 \\ +10 \\ \hline \end{array}$ • Identify a simple number sentence involving addition. ($81+2=$) • Solve addition problems within 50, involving “adding to,” and “putting together” using models or objects. • Describe or demonstrate + as putting together. • Identify the meaning of the + sign (can use terms such as: add, addition, plus, put together, combine) • Engage with the sign for +. • Engage with one and two-digit numbers.
<p>1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than</p>	<ul style="list-style-type: none"> • Describe the mental process used to subtract ten from a given number to get an answer. • Describe the mental process used to add ten to a given number to get an answer.

Grade-Level Standard Grade 1	Learning Progression <i>Building the Base & Engagement</i>
<p>the number, without having to count; explain the reasoning used.</p>	<ul style="list-style-type: none"> • Mentally subtract 10 from a given number up to 100 and state the number aloud. • Mentally add 10 to a given number up to 100 and state the number aloud. • Subtract ten objects from a given number by counting back using 1:1 correspondence and a number line as needed. • Add ten objects to a given number and count up using 1:1 correspondence. • Count backwards by ten from 100-10. • Point to and count groups of objects by ten up to 100. • Group objects by 10 up to 100. • Engage with a group of 10 objects representing the number ten.
<p>1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p>	<ul style="list-style-type: none"> • Describe the mental process used to subtract ten from a given number to get an answer.(i.e. place value, relationship between + and -, concrete models, etc.) • Mentally subtract 10 from a given number between 10- 90 and state the number aloud. • Subtract ten objects from a given number by counting back using 1:1 correspondence and a number line as needed. • Subtract ten objects by placing them on a number line from a given number and counting backward using 1:1 correspondence. • Count backwards by ten from 90-10. • Point to and count groups of objects by ten up to 90. • Group objects by 10 up to 90. • Engage with a group of 10 objects representing the number ten.
Measurement and Data	
<p>Measure lengths indirectly and by iterating length units. 1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.</p>	<ul style="list-style-type: none"> • Compare the measurements of two objects by discussing lengths. • Measure two objects and record lengths of each. • Measure an object with another object that is not typically used to measure length. • Measure an object with a standard tool used to measure length. • Select an object or measuring tool that can be used to measure length. • Select two objects to compare by length. • Place 3 objects in order from shortest to longest or longest to shortest. • Select 3 objects with varied lengths. • Engage with measuring tools used to determine length.
<p>1.MD.2 Express the length of an object as a whole number of length units by</p>	<ul style="list-style-type: none"> • Express the length of an object as a whole number.

Grade-Level Standard Grade 1	Learning Progression Building the Base & Engagement
<p>laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. <i>Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.</i></p>	<ul style="list-style-type: none"> • Measure the length of an object with multiple objects of the same size and record the length by object size. (i.e. measure a pencil using pennies; this pencil is 8 pennies long) • Line up measuring objects end-to-end to measure another larger object. • Select a set of non-traditional measuring tools to determine the length of a larger object. • Select an object to measure. • Engage with non-traditional measuring tools used to determine length.
<p>Work with time and money. 1.MD.3 Work with time and money. a. Tell and write time in hours and half-hours using analog and digital clocks. b. Identify pennies and dimes by name and value.</p>	<p>Time:</p> <ul style="list-style-type: none"> • Write time to the half-hour given the time verbally. • Read time to the half-hour on a digital clock. • Identify the hour and :30/half-hour display on a digital clock. • Tell time to the nearest half-hour on digital and analog clock by stating the full time. • Identify the number the minute hand is pointing to in the :30/half-hour position. • Identify the number the hour hand is pointing to on an analog clock. • Write time to the hour given the time verbally. • Read time to the hour on a digital clock. • Identify the hour and o'clock display on a digital clock. • Tell time to the nearest hour on digital and analog clock by stating both the hour and o'clock. • Identify the number the minute hand is pointing to in the o'clock position. • Identify the number the hour hand is pointing to in any of the o'clock positions. • Identify the long minute hand. • Identify the short hour hand. • Identify the hands on the face of the clock. • Identify the numbers on the face of the clock. • Engage with analog and/or digital clock. <p>Money:</p> <ul style="list-style-type: none"> • Sort coins by grouping pennies and dimes. • Identify the value of a dime. • Identify the value of a penny. • Identify the name of both a penny and a dime. • Engage with pennies and dimes. • Engage with money.


Grade-Level Standard Grade 1	Learning Progression Building the Base & Engagement
<p>Represent and interpret data. 1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.</p>	<ul style="list-style-type: none"> • Identify the total number of data points. • Identify the total number of data points in each category. • Ask questions about the data in the table. • Answer questions about the data in the table. (i.e. how many, more or less...) • Interpret data in a table by comparing all three columns. • Represent data by creating a table to organize information. • Organize data into up to a three column chart. • Fill in a chart or table using data. • Sort data by category. • Compile data from a survey to use in a chart or table. • Identify a data source. • Identify a chart or table. • Engage with a data chart.
Geometry	
<p>Reason with shapes and their attributes. 1.G.1 Distinguish between defining attributes, e.g., triangles are closed and three-sided, versus non-defining attributes, e.g., color, orientation, overall size; build and draw shapes that possess defining attributes.</p>	<ul style="list-style-type: none"> • Draw shapes with defining attributes. • Define attributes of a given shape. • Sort shapes by non-defining attributes (size, color, orientation). • Identify non-defining attributes. (i.e. color, orientation, overall size) • Match the term attribute to its definition. • Describe each shape in detail. (i.e. triangles have three sides and three angles) • Identify 2-D shapes. • Engage with 2-D shapes.
<p>1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three- dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. Students do not need to learn formal names such as "right rectangular prism."</p>	<ul style="list-style-type: none"> • Use 2-D shapes (such as attribute blocks or geoboards) to create a new shapes. • Compose 3-D shapes. (i.e.cubes, right rectangular prisms, right circular cones, and right circular cylinders) • Compose 2-D shapes. (i.e. rectangles, squares, trapezoids, triangles, diamond, half-circles, and quarter-circles) • Identify shapes as two-dimensional or three- dimensional (i.e., flat vs. solid). • Identify 3-D shapes by name. • Identify 2-D shapes by name. • Identify shapes vs.numbers or letters. • Engage with 3-D and 2-D shapes.

Grade-Level Standard Grade 1	Learning Progression <i>Building the Base & Engagement</i>
<p>1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words <i>halves</i>, <i>fourths</i>, and <i>quarters</i>, and use the phrases <i>half of</i>, <i>fourth of</i>, and <i>quarter of</i>. Describe the whole as two of or four of the shares in real-world contexts. Understand for these examples that decomposing into more equal shares creates smaller shares.</p>	<ul style="list-style-type: none"> • Describe each equal part as a share of the whole. • Partition circles into two or four equal parts. • Partition rectangles into two, four equal parts. • Identify the whole as “two halves,” “four fourths” etc. using real world examples. (i.e. pie, pizza, cookie, etc.) • Identify the parts as “halves,” “quarters,” “fourths”, etc. • Count the number of sections in a circle that has been divided into equal parts (e.g., half, quarter, third). • Use models to put together circles from equal fractional parts. • Use models to put together squares or rectangles from equal fractional parts. • Engage with fractional parts of a shape. • Engage with fractional shapes.

Grade 2

Grade-Level Standard Grade 2	Learning Progression <i>Building the Base & Engagement</i>
Operations and Algebraic Thinking	
<p>Represent and solve problems involving addition and subtraction. 2.OA.1 Use addition and subtraction within 100 to solve one- and two- step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Table 1, page 95.</p>	<ul style="list-style-type: none"> • Solve one and two step addition s and subtraction word problems within 50, involving situations where one is “adding to,” “taking from,” “putting together” and “taking apart,” using models or objects. • Solve addition and subtraction word problems within 20, involving situations where one is “adding to,” “taking from,” “putting together” and “taking apart,” using models or objects. • Solve addition and subtraction word problems within 10 involving situations where one is “adding to,” “taking from,” “putting together” and “taking apart,” using models or objects. • Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group • Understand that (+) symbol is addition (-) symbol is subtraction and (=) symbol is equal. • Engage with putting together and taking apart using models or objects. • Engage in 1:1 correspondence. • Engage in grouping like objects together.
<p>Add and subtract within 20. 2.OA.2 Fluently[®] add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. See standard 1.OA.6 for a list of mental strategies.</p>	<ul style="list-style-type: none"> • Understand associative and commutative properties • Understand that (+) symbol is addition (-) symbol is subtraction and (=) symbol is equal • Write the numbers 0-10. • Identify numbers 0-20. • Match numbers 0-20. • Engage with putting together and taking apart using models or objects • Engage in 1:1 correspondence • Interact in grouping like objects together. • Orally count, sign or count using assistive technology, with 1 second automaticity from 1-20. • Participate in fluency activities. (using assistive technology switches or voice output devices as needed) • Engage in fluency activities, songs, rhymes, tools. (numbers 1-20)
<p>Work with equal groups of objects to gain foundations for multiplication. 2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation</p>	<ul style="list-style-type: none"> • Identify the number of objects in a group (of up to 20) as odd or even. • Identify the number of objects in a group (of up to 10) as odd or even. • Represent a problem with groups of objects 4 x 2 e.g. 4 groups of 2 pencils. • Represent a number with group of objects e.g. 2 = 2 pencils. • Participate in grouping objects together by 2s. • Understand that (x) symbol is multiplication (-) and (=) symbol is equal. • Engage with putting together and taking apart using models or objects.

Grade-Level Standard Grade 2	Learning Progression <i>Building the Base & Engagement</i>
to express an even number as a sum of two equal addends.	<ul style="list-style-type: none"> Engage in 1:1 correspondence. Engage in grouping like objects together. Engage with like objects.
2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	<ul style="list-style-type: none"> Add the number of objects in an array with up to 5 rows and 4 columns, and represent in an equation. Add the number of objects in an array with up to 3 rows and 3 columns. Recognize a rectangular array. Write numbers to objects counted. Recognize that the term sum is related to an addition problem. Match objects counted with the corresponding number. Count objects. Arrange objects into like groups to show repeated addition. Understand that (+) symbol is addition and (=) symbol is equal. Engage with putting together models or objects. Engage in 1:1 correspondence Sort objects into like groups. Interact with with a group of like objects.
<i>Number and Operations Base Ten</i>	
Understand place value. 2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens - called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	<ul style="list-style-type: none"> Identify the correct, expanded form given a model or object representation of a two-digit number. Understand the position of the digit changes the place value Write numbers. Identify numbers Recognize 10 bundles of ones equals one hundred. Sort like objects into bundles of 10 ones. Sort like objects. Understand a two digit number represents tens and ones. Understand a one digit number represent ones. Interact with a bundle of objects. Engage with a objects.
2.NBT.2 Count forward and backward within 1,000 by ones, tens, and	<ul style="list-style-type: none"> Count to 100 by 1's, 5's, 10's Count backward sequentially.

Grade-Level Standard Grade 2	Learning Progression <i>Building the Base & Engagement</i>
<p>hundreds starting at any number; skip-count by 5s starting at any multiple of 5.</p>	<ul style="list-style-type: none"> Count forward sequentially. Represent a number of objects with a written numeral. Match a number of objects with a written numbers. Engage with real world counting of objects. Orally count, sign or count using assistive technology, with 1 second automaticity. Participate in fluency activities. (using assistive technology switches or voice output devices as needed) Engage in fluency activities, songs, rhymes, tools.
<p>2.NBT.3 Read and write numbers to 1,000 using base-ten numerals, number names, expanded form⁶, and equivalent representations, e.g., 716 is $700 + 10 + 6$, or $6 + 700 + 10$, or 6 ones and 71 tens, etc.</p>	<ul style="list-style-type: none"> Represent a number with 1's, 10's and 100's with objects or bundles of objects. Demonstrate by matching, drawing, or labeling what each digit in a number represents. Understand that expanded form is splitting numbers in to parts. Count to 100 by 1's, 5's, 10's Count backward sequentially. Count forward sequentially. Partition objects for counting. Represent a number of objects with a written numeral Match a number of objects with a written numbers Understand that (+) symbol is addition Interact with a group of objects to be counted Orally count, sign or count using assistive technology, with 1 second automaticity. Participate in fluency activities. (using assistive technology switches or voice output devices as needed) Engage in fluency activities, songs, rhymes, tools.
<p>2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p>	<ul style="list-style-type: none"> Compare two-digit numbers based on values of the digits in each place, using $>$, $=$, and $<$ symbols (e.g., $56 > 52$; $45 < 56$). Compare one-digit numbers, as shown in the picture below, based on values using $>$, $=$, and $<$ symbols (e.g., $6 > 2$; $5 < 6$).  <ul style="list-style-type: none"> The greater the number of digits the greater the number. Identify how many numbers are in the numbers to compare. Identify whether a set of objects is “greater than,” “less than” or “same as or equal to” another set of objects.

Grade-Level Standard Grade 2	Learning Progression <i>Building the Base & Engagement</i>
	<ul style="list-style-type: none"> • Understand place value as ones, tens, and hundreds. • Match sets of numbers to their respective number. • Match objects to a number. • Count objects. • Sort groups of objects. • Engage in math activities or games that practice the vocabulary “greater than,” “less than” or “same as or equal to” • Engage with sets of objects to compare. • Engage with objects that can be counted.
<p>Use place value understanding and properties of operations to add and subtract.</p> <p>2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>	<ul style="list-style-type: none"> • Add and subtract up to the sum of 50 using strategies based on place value (e.g., collecting the tens, collecting the ones, and composing ten ones to make a ten). • Add and subtract up to the sum of 20 using strategies based on place value. • Add and subtract up to the sum of 10 using models and concrete objects. • Count backward • Count forward • Understand associative and commutative properties • Understand that (+) symbol is addition (-) symbol is subtraction and (=) symbol is equal • Recognize that addition and subtraction of whole numbers are based on sequential counting. • Understand place value as ones, tens, and hundreds. • Understand that 10 bundles of ones equal 100. • Understand that 10 ones equal 1 ten in a bundle. • Write the numbers 0-100. • Identify numbers 0-100. • Match numbers 0-100. • Engage with putting together and taking apart using models or objects. • Engage in 1:1 correspondence. • Match a number to the objects. • Interact in grouping like objects together. • Count objects and groups of objects. • Interact with a group of objects to be counted. • Orally count, sign or count using assistive technology, with 1 second automaticity. • Participate in fluency activities. (using assistive technology switches or voice output devices as needed). • Engage in fluency activities, songs, rhymes, tools.

Grade-Level Standard Grade 2	Learning Progression <i>Building the Base & Engagement</i>
<p>2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.</p>	<ul style="list-style-type: none"> • Add 2 two-digit numbers using at least one strategy (e.g., concrete models or drawings, decomposing numbers, or strategies based on place value, properties of operations, and/ or relationships between addition and subtraction). • Add and subtract 2 two-digit numbers (multiples of 10) using at least one strategy (e.g., concrete models or drawings decomposing numbers, or strategies based on place value, properties of operations, and/ or relationships between addition and subtraction). • Add and subtract 1 two-digit number and 1 one- digit number, using models and concrete objects. • Add tens on their own. • Add ones on their own. • Understand that two digits is tens. • Understand that one digit is ones. • Understand that (+) symbol is addition and (=) symbol is equal • Engage with putting together and taking apart using models or objects • Engage in 1:1 correspondence. • Match a number to the objects. • Interact in grouping like objects together. • Engage with sets of objects. • Engage with numbers.
<p>Use place value understanding and properties of operations to add and subtract.</p> <p>2.NBT.7 Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; record the strategy with a written numerical method (drawings and, when appropriate, equations) and explain the reasoning used. Understand that in adding or subtracting three-digit numbers, hundreds are added or subtracted from hundreds, tens are added or subtracted from tens, ones are added or subtracted from ones;</p>	<ul style="list-style-type: none"> • Add 3 two-digit numbers using at least one strategy (e.g., concrete models or drawings, decomposing numbers, or strategies based on place value, properties of operations, and/ or relationships between addition and subtraction). • Add 2 two-digit numbers using at least one strategy (e.g., concrete models or drawings, decomposing numbers, or strategies based on place value, properties of operations, and/ or relationships between addition and subtraction). • Add and subtract 2 two-digit numbers (multiples of 10) using at least one strategy (e.g., concrete models or drawings decomposing numbers, or strategies based on place value, properties of operations, and/ or relationships between addition and subtraction). • Add and subtract 1 two-digit number and 1 one- digit number, using models and concrete objects. • Add 100's on their own • Add 10's on their own. • Add 1's on their own. • Demonstrate taking apart is subtraction and demonstrate putting together is adding • Understand that three digits is 100's. • Understand that two digits is 10's. • Understand that one digit is 1's.

Grade-Level Standard Grade 2	Learning Progression <i>Building the Base & Engagement</i>
<p>and sometimes it is necessary to compose or decompose tens or hundreds.</p>	<ul style="list-style-type: none"> • Understand that (+) symbol is addition (-) symbol is subtraction and (=) symbol is equal. • Count numbers 0-100 forward and backward. • Match subtract (-) to terms such as: take away, take apart, minus, subtract • Match add (+) to terms such as: count on, plus, put together, group. • Describe or demonstrate - as taking away using multi-sensory models. • Describe or demonstrate + as putting together using multi-sensory models. • Follow along and mimic “putting together” and “taking apart” as demonstrated using multi-sensory models to represent addition and subtraction. • Engage in demonstrations using multi-sensory models to represent addition and subtraction equations. • Engage in 1:1 correspondence. • Match a number to the objects. • Interact in grouping like objects together. • Engage with base 10 blocks. • Engage with concrete models and drawings.
<p>2.NBT.8 Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.</p>	<ul style="list-style-type: none"> • Add or subtract 100 to or from a given number. • Add or subtract 10 to or from a given number up to 100. • Add 100 to a given number with visual or physical representations. • Add 10 to a given number with visual or physical representations. • Partition and combine tens and ones • Partition small numbers • Count forward and backward by 100's to 1000. • Count forward and backward by 10's to 100. • Identify the 10's column on a 100 chart. • Identify the number 100. • Identify the number 10. • Engage with the numbers 10 or 100. • Orally count, sign or count using assistive technology, with 1 second automaticity. • Participate in fluency activities. (using assistive technology switches or voice output devices as needed). • Engage in fluency activities, songs, rhymes, tools.
<p>2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. Explanations may be supported by drawings or objects.</p>	<ul style="list-style-type: none"> • Identify or create a model that can be used to solve either an addition or a subtraction problem. • Identify or create a model that can be used to solve an addition problem. • Identify which model represents an addition problem, given a choice of two models. • Two digit equals 10's • Understand 1 digit equals 1's

Grade-Level Standard Grade 2	Learning Progression <i>Building the Base & Engagement</i>
	<ul style="list-style-type: none"> • Understand there are relationships among number • Count whole numbers sequentially • Write numbers • Match numbers to objects • Represent numbers with objects • Engage with numbers
Measurement and Data	
<p>Measure and estimate lengths in standard units.</p> <p>2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.</p>	<ul style="list-style-type: none"> • Identify numbers on a measuring tool. • Demonstrate lining up an object at 0. • Recognize the placement of the measuring tool when measuring the length starts at 0. • Identify 0 on a measuring tool as the starting point of an object to be measured. • Measure an object with a given tool such as a ruler, yardstick, meter stick or measuring tape. • Demonstrate lining up an object at 0. • Recognize the placement of the measuring tool when measuring the length starts at 0. • Identify 0 on a measuring tool as the starting point of an object to be measured. • Understand that length is the size of an object. • Match an object to measured with the closest in size measuring tool. • Match the name of the measuring tool to the tool. • Interact with various sized objects that can be measured. • Interact with with a given tool such as a ruler, yardstick, meter stick or measuring tape.
<p>2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.</p>	<ul style="list-style-type: none"> • Order objects based on their length. • Identify numbers on a measuring tool. • Measure an object with a given tool such as a ruler, yardstick, meter stick or measuring tape. • Recognize the vocabulary longer, shorter, about the same size. • Demonstrate lining up an object at 0. • Recognize the placement of the measuring tool when measuring the length starts at 0. • Understand that length is the size of an object. • Match an object to measured with the closest in size measuring tool. • Match the name of the measuring tool to the tool. • Interact with various sized objects that can be measured. • Interact with with a given tool such as a ruler, yardstick, meter stick or measuring tape.

Grade-Level Standard Grade 2	Learning Progression <i>Building the Base & Engagement</i>
2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.	<ul style="list-style-type: none"> • Match the unit of length to the tool. • Measure an object using non-standards units. (e.g., same length pencils, paperclips). • Identify which measurement tool to use (e.g., use a ruler to measure length; use a scale to find weight) • Interact with objects to be measured. • Interact with measuring tools.
2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard-length unit.	<ul style="list-style-type: none"> • Understand to get the difference in length between two objects what operation is used. • Measure and compare the length of an object with two different standard or nonstandard tools. • Measure to determine how much longer one object is than another using a non-standard unit. • Demonstrate lining up an object at 0. • Identify 0 on a measuring tool as the starting point of an object to be measured. • Identify numbers on a measuring tool • Understand the vocabulary longer and shorter • Order objects by their size • Interact with a measuring tool • Interact with objects to be measured
Relate addition and subtraction to length. 2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same whole number units, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)	<ul style="list-style-type: none"> • Solve addition and subtraction word problems involving lengths. • Solve addition word problems involving length. • Set-up a equation from a given word problem. • Measure the length of two objects using non- standard units to determine the total length of the two objects combined. • Identify the operation used when interpreting the word problem. • Recognize that words can be represented by numbers. • Match subtract (-) to terms such as: take away, take apart, minus, subtract • Match add (+) to terms such as: count on, plus, put together, group. • Describe or demonstrate - as taking away using multi-sensory models. • Describe or demonstrate + as putting together using multi-sensory models. • Follow along and mimic “putting together” and “taking apart” as demonstrated using multi-sensory models to represent addition and subtraction. • Engage in demonstrations using multi-sensory models to represent addition and subtraction equations. • Interact with a word problem. • Interact with a drawing.
2.MD.6 Represent whole numbers as lengths from 0 on a number line	<ul style="list-style-type: none"> • Add or subtract using a number line • Count on a number line

Grade-Level Standard Grade 2	Learning Progression Building the Base & Engagement
<p>diagram^o with equally spaced points corresponding to the numbers 0, 1, 2,..., and represent whole-number sums and differences within 100 on a number line diagram.</p>	<ul style="list-style-type: none"> • Identify 0 on a number line • Demonstrate that moving forward is addition and moving backwards is subtraction on a number line. • Identify numbers on a number line. • Interact with a number line.
<p>Work with time and money. 2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.</p>	<ul style="list-style-type: none"> • Tell time to the nearest 15 minute intervals on digital and analog clocks. • Tell time to the nearest hour and half-hour intervals on digital and analog clocks. • Tell time to the nearest hour on digital and analog clocks. • Identify events that happen in the morning (a.m.) or afternoon/evening (p.m.). • Match an event to the time of the day that it might happen in the morning (a.m.) or afternoon/evening (p.m.). • Match (a.m.) to the morning and (p.m.) to the night • Interact with a clock
<p>2.MD.8 Solve problems with money. a. Identify nickels and quarters by name and value. b. Find the value of a collection of quarters, dimes, nickels, and pennies. c. Solve word problems by adding and subtracting within 100, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the \$ and ¢ symbols appropriately (not including decimal notation).</p>	<ul style="list-style-type: none"> • Identify coins/bills and match to their values and corresponding symbol (\$, ¢). • Solve problems involving a combination of coins and dollar bills within a word problem • Identify the operation used when interpreting the word problem. • Recognize that words can be represented by numbers. • Understand that (+) symbol is addition (-) symbol is subtraction and (=) symbol is equal • Interact with a word problem. • Count by 1's, 5's, 10's. • Sort like coins together • Match like coins and one-dollar bills. • Select the correct coins and/or bills to match a given amount. • Identify coins (quarters, dimes, nickels, pennies). • Interact with coins and dollars.
<p>Represent and interpret data. 2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object. Show the measurements by creating a line plot^o, where the horizontal scale is marked off in whole-number units.</p>	<ul style="list-style-type: none"> • Gather data (e.g., measure the length of an object) and graph the data on a line plot. • Graph given measurement data on a line plot. • Interact with a line plot. • Identify numbers on a number line. • Demonstrate lining up an object at 0. • Identify 0 on a measuring tool as the starting point of an object to be measured. • Identify numbers on a measuring tool. • Understand the vocabulary longer and shorter. • Order objects by their size.


Grade-Level Standard Grade 2	Learning Progression <i>Building the Base & Engagement</i>
	<ul style="list-style-type: none"> • Explore objects with different measurements. • Interact with a measuring tool. • Interact with objects to be measured tactually or visually.
Represent and interpret data. 2.MD.10 Organize, represent, and interpret data with up to four categories; complete picture graphs when single-unit scales are provided; complete bar graphs when single-unit scales are provided; solve simple put-together, take-apart, and compare problems in a graph. See Table 1, page 95.	<ul style="list-style-type: none"> • Create a bar or picture graph, with given data and answer questions about the graph using a single unit scale. • Create a picture graph, given data and answer questions about the graph. • Classify and count objects in categories of a data set (e.g., given a set of colored cubes: identify if any are red, count how many are red, etc.). • Demonstrate putting together and taking apart of objects. • Interact with data. • Match parts of a graph to their function. • Identify parts of a graph. • Interact with a graph tactually or visually.
Geometry	
Reason with shapes and their attributes. 2.G.1 Recognize and identify triangles, quadrilaterals, pentagons, and hexagons based on the number of sides or vertices. Recognize and identify cubes, rectangular prisms, cones, and cylinders.	<ul style="list-style-type: none"> • Classify shapes by their defining attributes (e.g., quadrilaterals, triangles, number of sides and angles) • Identify cubes, rectangular prisms, cones, cylinders and spheres. • Identify cubes, rectangular prisms, cones, cylinders and spheres. • Identify three-dimensional shapes in the environment. • Identify shapes as two-dimensional or three-dimensional (i.e., flat vs. solid). • Name objects from the environment that are 2D and 3D shapes. • Select an object from the environment and match to a description. • Explore and manipulate shapes tactually or visually.
2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	<ul style="list-style-type: none"> • Partition rectangles into two, three or four equal parts; identify the parts as “halves,” “thirds,” “quarters,” “half of,” “a third of,” or “a quarter of;” and identify the whole as “two halves,” “three thirds,” “four fourths” or “four quarters.” • Recognize the vocab: <ul style="list-style-type: none"> ○ Equal to: The same value as. ○ Fourth: One of four equal parts. ○ Fraction: Part of a whole. ○ Half Circle: One of two equal parts of a circle. ○ Half/Halves: One or more of two equal parts of a whole. ○ Quarter Circle: One of four equal parts of a circle. ○ Quarter of: One of four equal parts

Grade-Level Standard Grade 2	Learning Progression <i>Building the Base & Engagement</i>
	<ul style="list-style-type: none"> Count the number of sections in a rectangle that has been divided into equal parts (e.g., half, quarter, third). Demonstrate counting columns from left to right. Demonstrate understanding of same size. Match the name of the rectangle with its shape. Name a rectangle and circle from the environment. Select a rectangle and circle from the environment to describe. Interact with real world objects that are rectangles tactually or visually.
<p>2.G.3 Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words <i>halves</i>, <i>thirds</i>, or <i>fourths</i> and <i>quarters</i>, and use the phrases <i>half of</i>, <i>third of</i>, or <i>fourth of</i> and <i>quarter of</i>. Describe the whole as two halves, three thirds, or four fourths in real-world contexts. Recognize that equal shares of identical</p>	<ul style="list-style-type: none"> Partition rectangles into two, three or four equal parts; identify the parts as “halves,” “thirds,” “quarters,” “half of,” “a third of,” or “a quarter of;” and identify the whole as “two halves,” “three thirds,” “four fourths” or “four quarters.” Partition circles into two or four equal parts; identify the parts as “halves,” “quarters,” “half of,” “a third of” or “a quarter of;” and identify the whole as “two halves,” “three thirds,” “four fourths” or “four quarters.” Recognize the vocab: <ul style="list-style-type: none"> Equal to: The same value as. Fourth: One of four equal parts. Fraction: Part of a whole. Half Circle: One of two equal parts of a circle. Half/Halves: One or more of two equal parts of a whole. Quarter Circle: One of four equal parts of a circle. Quarter of: One of four equal parts. Match a model with “halves,” “thirds,” “fourths” or “quarters.” Understand that smaller shapes are made when dividing shapes into parts Count the number of sections in a circle or rectangle that has been divided into equal parts (e.g., half, quarter, third). Demonstrate counting columns from left to right.


Extended Standards with Learning Progressions for Grades 3 - 8

Grade 3


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←				→ Least Complex
Operations and Algebraic Thinking				
<i>Represent and solve problems involving multiplication and division.</i>				
3.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. (Note: These standards are written with the convention that $a \times b$ means a groups of b objects each; however, because of the commutative property, students may also interpret 5×7 as the total number of objects in 7 groups of 5 objects each.)	3.OA.1a Represent products of whole numbers up to 10×10 using arrays, area models, or physical objects (whole numbers 0 through 10).	3.OA.1b Represent products with factors of 1s, 2s, 3s, 4s, 5s, and 10s using arrays, area models, or physical objects (whole numbers 1 through 10).	3.OA.1c Represent products with factors of 1s, 2s, and 5s using arrays, area models, or physical objects (whole numbers 1 through 10).	<ul style="list-style-type: none"> Identify 1, 2, and 5 blocks. Identify groups of blocks 1s, 2s, and 5s. Build groups of blocks into rows and columns (arrays). Count the number of blocks in a given array. Build an array and count the number of blocks. Skip count by 2s and 5s. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns. Identify the number of blocks in each row and each column. Match an array to its factors. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with physical objects (blocks).
3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. <i>For</i>	3.OA.2a Represent quotients of single-digit whole numbers up to 100.	3.OA.2b Represent quotients using arrays, area models, or other physical representations for whole number factors of 1s, 2s, 3s, 4s, 5s,	3.OA.2c Represent quotients using arrays, area models, or other physical representations for whole numbers	<ul style="list-style-type: none"> Identify 1, 2, and 5 blocks. Identify groups of blocks 1s, 2s, and 5s. Share up to 10 objects equally between 2 and 5 people (without remainders). Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with physical objects (blocks).


				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex 				Least Complex
<i>example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.</i>		and 10s with products not exceeding 100.	factors of 1s, 2s, and 5s with products not exceeding 10, 20, and 50, respectively.	
3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)	3.OA.3a Solve word problems with products and/or quotients of whole numbers using arrays, area models, or other physical representations (whole numbers factors of 0 through 10).	3.OA.3b Solve word problems with products of whole numbers 1s, 2s, 3s, 4s, 5s, and 10s using arrays, area models, or other physical objects (products not exceeding 100).	3.OA.3c Represent word problems with products of whole number factors of 1s, 2s, and 5s using arrays, area models or other physical representations (whole numbers 1 through 10).	<ul style="list-style-type: none"> Identify the appropriate numbers in the multiplication word problem. Match a physical representation or drawing to the multiplication word problem. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns. Write a number sentence to express the total as a sum of equal addends. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings representing multiplication word problems.
3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times \square = 48$; $5 = \square \div 3$; $6 \times 6 = \square$.	3.OA.4a When given a physical or visual model representing a multiplication fact (whole number factors of 0 through 10 with products not exceeding 100) and a set of 3 answer choices, identify the	3.OA.4b When given a physical or visual model representing a multiplication fact (whole number factors of 1s, 2s, 3s, 4s, 5s, and 10s with products not exceeding 100) and a set of 3 answer choices, identify the unknown whole number.	3.OA.4c Match a provided physical or visual model to one of three provided multiplication or division number sentences. AND When given a number sentence, identify the	<ul style="list-style-type: none"> Identify a number sentence. Identify a physical object or drawing. Know that a symbol χ can represent a missing (unknown) number. Know the operations for each of the three symbols (\times, $=$, \div) Read and interpret a traditional number sentence ($2 \times 2 = 4$) Relate a picture or objects to a number sentence. Group or partition sets of numbers into equal groups to determine the missing number.

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←				→ Least Complex
	unknown whole number.		operations symbol for \div , \times , and $=$.	<ul style="list-style-type: none"> Record a number sentence to express the total as the sum of equal addends. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings that represents an equation.
Understand properties of multiplication and the relationship between multiplication and division.				
3.OA.5 Apply properties of operations as strategies to multiply and divide. For example, if $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known (Commutative Property of Multiplication); $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$ (Associative Property of Multiplication); knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$ (Distributive Property). Students need not use formal terms for these properties.	3.OA.5a Physically or visually solve multiplication or division number sentences (whole number factors of 0 through 10 with products not exceeding 100) using the commutative and/or distributive properties (e.g., solving 3×8 by adding 3×5 to 3×3).	3.OA.5b Physically or visually solve multiplication number sentences (whole number factors of 1s, 2s, 3s, 4s, 5s, and 10s with products not exceeding 100) using the commutative property.	3.OA.5c Physically or visually match multiplication number sentences using the commutative property.	<ul style="list-style-type: none"> Identify a number sentence. Count the number of objects in an array. Recognize the symbols for multiplication (\times) and equals ($=$). Read and interpret a traditional number sentence ($2 \times 3 = 6$). Relate a picture or objects to a number sentence. Recognize factors in an array. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns. Write the number sentence to express the total as a product of two factors. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings that represents an equation.
3.OA.6 Understand division as an unknown factor problem. For	3.OA.6a Understand that	3.OA.6b Understand division as the	3.OA.6c Show division as sorting	<ul style="list-style-type: none"> Identify a number sentence. Count the number of objects in an array.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex 				Least Complex
example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.	the inverse operation of division is multiplication. Can answer a multiplication question to solve for division (What times 3 equals 15?). Then solve the division problem (understand that 3 groups of 5 equals 15).	inverse operation of multiplication by sorting objects or pictures into equal groups and matching multiplication/division problems ($3 \times 5 = 15$ $15 \div 3 = 5$).	objects or pictures into equal groups.	<ul style="list-style-type: none"> Recognize the symbols for division (\div) and equals ($=$). Read and interpret a traditional number sentence ($6 \div 2 = 3$). Relate a picture or objects to a number sentence. Recognize factors in an array. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings that represents an equation.
<i>Multiply and divide within 100.</i>				
3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division, e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$, or properties of operations. Limit to division without remainders. By the end of grade 3, know from memory all products of two one-digit numbers.	3.OA.7a Fluently know all products (whole number factors of 0 through 10) and their respective division problems.	3.OA.7b Fluently know all products for whole number factors of 1s, 2s, 3s, 4s, 5s, and 10s with products not exceeding 100.	3.OA.7c Solve multiplication number sentences for multiples of 1s, 2s, and 5s (whole numbers 1 through 10) using arrays, area models, or other physical representations.	<ul style="list-style-type: none"> Identify a number sentence. Count the number of objects in an array. Recognize the symbols for multiplication (\times) and equals ($=$). Read and interpret a traditional number sentence ($2 \times 3 = _$). Relate a picture or objects to a number sentence. Know that a symbol χ can represent a missing product. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings that represents a multiplication number sentence.
<i>Solve problems involving the four operations, and identify and explain patterns in arithmetic.</i>				

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter or a symbol, which stands for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole-number answers. Students may use parentheses for clarification since algebraic order of operations is not expected.	3.OA.8a Represent a 2-step problem using an equation with a symbol (e.g., shape) standing for the unknown and solve.	3.OA.8b Identify the array, area model, or other physical representation that shows the solution of a 1-step number sentence from a word problem (excludes division).	3.OA.8c Identify the number sentence that correlates with a given 1-step word problem (excludes division).	<ul style="list-style-type: none"> Identify a number sentence. Count the number of objects in an array. Recognize the symbols for addition (+), subtraction (−), multiplication (×), and equals (=). Read and interpret a traditional one-step number sentence ($2 \times 3 = \underline{\quad}$). Relate a picture or objects to a number sentence. Know that a symbol χ can represent a missing value. <p>Engagement Statements (<i>demonstration of engaged in the topic</i>)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings representing addition, subtraction, or multiplication word problems.
3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. <i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i>	3.OA.9a Identify and explain arithmetic patterns in a number chart or addition and multiplication tables.	3.OA.9b Identify arithmetic patterns in a number chart, or addition and multiplication tables.	3.OA.9c Use odd or even numbers to identify/make a pattern using repeated addition within a 100s chart.	<ul style="list-style-type: none"> Identify the numerals 1-20 on a 100s chart. Know the word names for the numbers 1-100. Count from 1-100. Write numerals from 0 to 20. Represent a number of objects with a written numeral 0-20. Count the number of objects up to 20. Skip count by 2s and 5s up to 20 using a physical objects and visual models. Repeatedly add the same number using physical objects and visual models. Relate counting to addition by counting on 2 to add 2 or 5 to add 5. <p>Engagement Statements (<i>demonstration of engaged in the topic</i>)</p>

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				<ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings (may include 100s chart) representing whole numbers within 20.
Number and Operations in Base Ten				
<i>Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of strategies and algorithms may be used.</i>				
3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.	3.NBT.1a Use place value understanding and a physical and/or visual representation to round multi-digit whole numbers to the nearest 10 or 100.	3.NBT.1b Identify a given number to the nearest 10s place when using number lines and/or number grids (e.g., 22 will round to 20).	3.NBT.1c Using a physical or visual representation for numbers 0 through 10, when shown two numbers, show which number is closer to 0 or 10 (e.g., shown 5 or 6, student is asked which number shown is closer to 10).	<ul style="list-style-type: none"> Know what a number line is. Know the order of the numbers from 0 to 10. Identify a whole number on a number line marked with whole numbers up to 10. Identify 0 on a number line. Identify a missing whole number value on a number line marked with whole number up to 10. Compare distances of objects using a vertical or horizontal number line. Understand that 2 is the distance from 0 to 2 and 3 is the distance from 0 to 3 using standard units for all lengths from 1 to 10. <p>Engagement Statements (<i>demonstration of engaged in the topic</i>)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings (may include 100s chart) representing whole numbers within 20.
3.NBT.2 Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	3.NBT.2a Add and subtract within 500 using strategies based on place value, and the relationship between addition	3.NBT.2b Add and subtract within 100 using strategies based on place value, and the relationship between addition and subtraction (no calculator).	3.NBT.2c Add and subtract within 20 using strategies based on place value, and the relationship between addition and subtraction	<ul style="list-style-type: none"> Represent a number with a set of physical objects or a drawing. Understand addition is the combining of two (or more) sets of objects. Understand subtraction is taking away of one amount of objects from another. Understand that addition and subtraction are opposites.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex 				Least Complex
	and subtraction (no calculator).		(no calculator, but could include concrete objects or number charts).	<ul style="list-style-type: none"> Know the symbols for addition (+), subtraction (−), and equals (=). Relate counting to addition and subtraction, e.g., by counting on 2 to add 2. Add and subtract within 10 using strategies. Strategies may include: <ul style="list-style-type: none"> Counting on Making ten ($8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$) Decomposing a number leading to a ten ($13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$) Using the relationship between addition and subtraction; knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$ and Creating equivalent but easier or known sums (adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$). <p>Engagement Statements (<i>demonstration of engaged in the topic</i>)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings (may include 100s chart) representing whole numbers within 20.
3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range of 10–90, e.g., 9×80 , 5×60 , using strategies based on place value and properties of operations.	3.NBT.3a Multiply one-digit whole numbers by multiples of 10 using visual and/or physical representation.	3.NBT.3b Multiply one-digit whole numbers by 10 (e.g., $3 \times 10 = 30$).	3.NBT.3c When shown a number sentence of one-digit whole number multiplied by 10, match the product to the number sentence when	<ul style="list-style-type: none"> Count to 10. Count to 10 using objects. Create multiple groups of 10 using objects. Repeatedly add groups of 10 using physical objects. Relate counting to addition by counting on 10 to add 10.


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←				→ Least Complex
			shown 2 possible products (e.g., $5 \times 10 = 50$ or 80).	<ul style="list-style-type: none"> Know the symbols for multiplication (\times) and equals ($=$). Relate multiplication to repeated addition by writing a number sentence. Represent a number with a set of physical objects or a drawing. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings (100s chart or multiplication chart).
Number and Operations – Fractions				
<i>Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.</i>				
3.NF.1 Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.	3.NF.1a Match fractions with their model (limit to fractions with denominators of 2, 3, 4, 6, 8).	3.NF.1b Match fractions with their model (limit to $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$).	3.NF.1c Identify a unit fraction ($\frac{1}{4}$ or $\frac{1}{2}$) as part of a whole when shown as a physical and/or visual representation.	<ul style="list-style-type: none"> Identify a whole partitioned into 2 or 4 equal shares. Describe the equal shares of a whole as <i>halves</i> or <i>fourths</i>, or <i>half of</i> or <i>a fourth of</i>. Describe the whole as two halves or four fourths. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with fraction models.
3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the	3.NF.2a Identify fractions on a number line marked in equal parts matching the fraction denominator (limit to fractions with denominators of 2, 3, 4, 6, 8).	3.NF.2b Identify fraction(s) on a number line marked in equal parts matching the fraction(s)' denominator (limit to denominators of 2, 3 and 4).	3.NF.2c Identify a fraction on a number line marked in equal parts matching the fraction denominator (limit to $\frac{1}{2}$ and $\frac{1}{4}$).	<ul style="list-style-type: none"> Know what a number line is. Identify 0 on a number line. Know each marking on the number line represents a unit fraction of $\frac{1}{4}$ or $\frac{1}{2}$. Identify the fractional markings on a number line marked with fractions of $\frac{1}{4}$ or $\frac{1}{2}$. Identify a missing fraction on a number line marked with $\frac{1}{4}$ or $\frac{1}{2}$ units up to 1. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none">

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←				→ Least Complex
<p>endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.</p> <p>b. Represent a fraction $\frac{a}{b}$ (which may be greater than 1) on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.</p>				<ul style="list-style-type: none"> Interact with number lines with fractional unit markings of $\frac{1}{4}$ or $\frac{1}{2}$.
<p>3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p>a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.</p> <p>b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.</p> <p>c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.</p> <p>d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that</p>	<p>3.NF.3a Use a visual fraction model to identify greater than, less than, and equal to when comparing 2 fractions.</p>	<p>3.NF.3b Use visual fraction models to identify equivalent fractions with denominators of 2, 4, 6, and 8.</p>	<p>3.NF.3c Identify equivalent fractions of $\frac{1}{2}$ and $\frac{1}{4}$ when represented with visual fraction models (e.g. matching model of $\frac{1}{2}$ and $\frac{2}{4}$ on a number line).</p>	<ul style="list-style-type: none"> Identify the same sized whole partitioned into 2 and 4 equal shares. Describe the equal shares of a whole as <i>one half of</i> or <i>two fourths of</i>. Describe the whole as two halves or four fourths. <p>Engagement Statements (<i>demonstration of engaged in the topic</i>)</p> <ul style="list-style-type: none"> Interact with area (rectangles) and length (number lines) fraction models.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←				→ Least Complex
comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.				
Measurement and Data				
<i>Solve problems involving money, measurement, and estimation of intervals of time, liquid volumes, and masses of objects.</i>				
3.MD.1 Work with time and money. a. Tell and write time to the nearest minute. Measure time intervals in minutes (within 90 minutes). Solve real-world problems involving addition and subtraction of time intervals (elapsed time) in minutes, e.g., by representing the problem on a number line diagram or clock. b. Solve word problems by adding and subtracting within 1,000, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the \$ and ¢ symbol appropriately (not including decimal notation).	3.MD.1a1 Tell time to the nearest 15 minutes on an analog clock. AND 3.MD.1a2 Name and/or identify equivalent combinations of coins and/or bills.	3.MD.1b1 Tell time to the nearest 30 minutes on an analog clock. AND 3.MD.1b2 Identify, name, and state value for all coins and bills (coins: pennies, nickels, dimes, quarters; bills: \$1, \$5, \$10, \$20).	3.MD.1c1 Tell time to the nearest hour on an analog clock. AND 3.MD.1c2 Identify and name all coins and bills.	<ul style="list-style-type: none"> Count to 12. Tell time using a digital clock. Know the meaning of the hour and the minute hands on an analog clock. Count to 12 using an analog clock. Read the hour hand on an analog clock at different times of a day. Describe differences between U.S. coins. Describe differences between U.S. bills. Find numerals on U.S. bills (\$1, \$5, and \$10). Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with a clock and U.S. currency (pennies, nickels, dimes, quarters, \$1, \$5, and \$10).
3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. Add, subtract, multiply, or divide whole numbers to solve one-step word	3.MD.2a Solve 1-step word problems involving measures of liquid volumes and	3.MD.2b Using models and drawings, measure and estimate liquid volumes and masses of objects using	3.MD.2c Select the appropriate tool to measure volume and mass (e.g., measuring cup, scale).	<ul style="list-style-type: none"> Describe measurable attributes of a single object using terms such as heavy/light. Directly compare two measuring cups to see which one holds more and which one holds less. Directly compare two beakers to see which one holds more and which one holds less.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←				→ Least Complex
problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems involving notions of "times as much"; see Table 2, page 96.	masses of objects using standard units of measure.	standard units of measure (e.g., measuring cup, scale).		<ul style="list-style-type: none"> Use different types of scales to measure mass of objects. Use different types of cups to measure volume of liquids. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with measurement tools for volume and mass.
<i>Represent and interpret data.</i>				
3.MD.3 Create scaled picture graphs to represent a data set with several categories. Create scaled bar graphs to represent a data set with several categories. Solve two-step "how many more" and "how many less" problems using information presented in the scaled graphs. <i>For example, create a bar graph in which each square in the bar graph might represent 5 pets, then determine how many more/less in two given categories.</i>	3.MD.3a Create scaled bar (or picture) graph from given or collected data sets and interpret the graph, including solving 1-step (e.g., "how many more" "how many less" problems).	3.MD.3b Identify quantities from a picture or bar graph (e.g., in a class graph representing pets, represent 4 cats with 4 blocks or 4 cat pictures and 2 hamsters with 2 blocks or pictures).	3.MD.3c Sort data on a bar graph (e.g., weather—sunny, cloudy, rainy, snowy)	<ul style="list-style-type: none"> Classify objects into categories. Count the number of objects in each category. Sort U.S. currency by coins (pennies, nickels, dimes, quarters) or bills (\$1, \$5, \$10). Create a bar graph with a scale of 1 by stacking physical objects. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with a bar graph.
3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by creating a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	3.MD.4a Measure objects using a ruler to the nearest fourth of an inch.	3.MD.4b Measure objects using a ruler to the nearest half inch.	3.MD.4c Measure objects using a ruler to the nearest inch.	<ul style="list-style-type: none"> Identify inches on a ruler. Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end. Understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none">

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←				→ Least Complex
				<ul style="list-style-type: none"> Interact with a ruler.
3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.	3.MD.5a Recognize the number of units in a given surface area represents an array multiplication problem.	3.MD.5b Understand that an equal-sided square can represent 1 unit of measure and can be counted to determine the area of a plane figure.	3.MD.5c Understand that the term “area” is related to measurement of a surface.	<ul style="list-style-type: none"> Identify surfaces where an area can be measured. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with flat two-dimensional surfaces.
3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	3.MD.6a Find the area of rectangles with whole-number side lengths by counting unit squares (limit area up to 40).	3.MD.6b Find the area of rectangles with whole-number side lengths by counting unit squares (limit to factors of 1s, 2s, 3s, 4s, 5s, and 10s with products not exceeding 30).	3.MD.6c Find the area of rectangles with whole-number side lengths by counting unit squares (limit factors of 1s, 2s, and 5s and areas up to 20).	<ul style="list-style-type: none"> Identify surfaces where an area can be measured. Lay unit squares up to 20 on a flat surface. Count unit squares up to 20 on a flat surface. Arrange unit squares up to 20 into rows and columns. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with flat two-dimensional surfaces.
3.MD.7 Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-	3.MD.7a Given the side length measures for a rectangle, find the area (whole number factors with areas limited to 40).	3.MD.7b Given a visual model of a tiled rectangle, identify a number sentence (repeated addition or multiplication) that represents a solution for finding the area (whole number	3.MD.7c Use tiling to cover the area of a square and count the tiles (unit squares) to find the area (whole number factors with areas limited to 20).	<ul style="list-style-type: none"> Identify surfaces where an area can be measured. Lay unit squares up to 20 on a flat surface. Count unit squares up to 20 on a flat surface. Arrange unit squares up to 20 into rows and columns. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with flat two-dimensional surfaces.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex 				Least Complex
number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$ (represent the distributive property with visual models including an area model). d. Recognize area as additive. Find the area of figures composed of rectangles by decomposing into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.		factors with areas limited to 30).		
<i>Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures</i>				
3.MD.8 Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	3.MD.8a Solve one-step measurement word problems involving shapes with the same area and different perimeters.	3.MD.8b Solve addition or subtraction measurement word problems involving perimeter.	3.MD.8c Solve addition measurement problems by finding the perimeter of a rectangle represented on a grid.	<ul style="list-style-type: none"> Identify surfaces where a perimeter can be measured. Lay inch squares up to 20 around a flat surface. Count inch squares up to 20 all the way around a flat surface. Measure length and width with units and record the measurement. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with flat two-dimensional surfaces.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
Geometry				
<i>Reason with shapes and their attributes.</i>				
3.G.1 Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles).	3.G.1a Sort quadrilaterals by the number of sides and/or the presence or absence of square corners (right angles) (limit quadrilaterals to rectangles, squares, and rhombuses).	3.G.1b. Sort polygons with up to 8 sides by the number of sides (Limit quadrilaterals to rectangles, squares and rhombuses).	3.G.1c Match objects in the environment to their two-dimensional shape based on the number of sides (e.g., match a stop sign in the real world to an octagon shape).	<ul style="list-style-type: none"> Identify shapes as two-dimensional (lying in a plane, flat) or three-dimensional (solid). Name shapes regardless of their orientations or overall size. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with two- and three-dimensional objects in their environment.
3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.	3.G.2a Partition rectangles into two, three, or four equal parts; identify a part as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$.	3.G.2b Partition rectangles into two or four equal parts, identify the parts as “halves,” “quarters,” and whole.	3.G.2c Count the number of sections in a rectangle that has been divided into equal parts (limit to half and quarter).	<ul style="list-style-type: none"> Identify a whole partitioned into 2 or 4 equal shares. Describe the equal shares of a whole as <i>halves</i> or <i>fourths</i>, or <i>half of</i> or <i>a fourth of</i>. Describe the whole as two halves or four fourths. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with fraction models.


Grade 4

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
Operations and Algebraic Thinking				
<i>Use the four operations with whole numbers to solve problems.</i>				
4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	4.OA.1a Solve a multiplicative comparison using a visual and/or physical representation (limited to multiples of 2s, 3s, 4s, 5s, 6s, and 10s).	4.OA.1b Solve a multiplicative comparison problem using linear or other physical representations (limit to whole number factors of 2s, 4s, 5s, and 10s).	4.OA.1c Solve a multiplicative comparison problem using linear models or other physical representations (limit to whole number factors of 2s and 5s).	<ul style="list-style-type: none"> Identify 2 and 5 blocks. Identify groups of blocks 2s and 5s. Build groups of blocks into rows and columns (arrays). Count the number of blocks in a given array. Build an array and count the number of blocks. Skip count by 2s and 5s. Identify the number of blocks in each row and each column. Match an array to its factors. Listen to uses of multiplicative comparisons (<i>times as much</i> or <i>times as many</i>) in real-world contexts. (Ohio Learning Standards for Mathematics, pg. 96) Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with linear models or physical objects (blocks).
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. Drawings need not show details, but should show the	4.OA.2a Solve for the unknown product or quotient when given a word problem involving a multiplicative comparison (limit to whole number factors to 10×10).	4.OA.2b Given an array, area model, or other physical model that best represents a multiplicative comparison in a word problem, solve for the unknown product or quotient	4.OA.2c Identify the array, area model, or other physical representation that best represents a multiplicative comparison in a word problem	<ul style="list-style-type: none"> Identify 2 and 5 blocks. Identify groups of blocks 2s and 5s. Build groups of blocks into rows and columns (arrays). Count the number of blocks in a given array. Build an array and count the number of blocks. Skip count by 2s and 5s. Identify the number of blocks in each row and each column.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)		(limit to whole number factors of the 2s, 4s, 5s, and 10s).	(limit to multiples of 2s and 5s).	<ul style="list-style-type: none"> Match an array to its factors. Read multiplicative comparison statements (<i>times as much</i> or <i>times as many</i>) in real-world contexts. (Ohio Learning Standards for Mathematics, pg. 96) Solve 1-step number sentences involving multiplication or division. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with linear models or physical objects (blocks).
4.OA.3 Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	4.OA.3a Determine the operation and correctly solve one-step word problems with remainders when given visual and/or physical representations (whole numbers within 1,000).	4.OA.3b Determine the operation(s) and correctly solve two-step word problems, without remainders, when given visual and/or physical representations (whole numbers; sums to 100).	4.OA.3c Solve a one-step word problem using a given visual and/or physical model (whole numbers; sums to 30; factors of 1s, 2s, 5s, and 10s).	<ul style="list-style-type: none"> Identify a number sentence. Count the number of objects in an array. Recognize the symbols for addition (+), subtraction (−), multiplication (×), division (÷), and equals (=). Read and interpret a traditional one-step number sentence ($2 \times 3 = \square$). Relate a picture or objects to a number sentence. Know that a symbol \square can represent a missing value. Count to 30. Count physical objects up to 30. Identify groups of blocks 2s, 5s, and 10s. Build groups of blocks into rows and columns (arrays). Count the number of blocks in a given array. Build an array and count the number of blocks. Identify the number of blocks in each row and each column. Match an array to its factors. Engagement Statements (<i>demonstration of engaged in the topic</i>)

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings representing addition, subtraction, or multiplication word problems.
<i>Gain familiarity with factors and multiples.</i>				
4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	4.OA.4a Using a multiplication table or other tool, identify the factor pairs for whole numbers in the range of 1–50.	4.OA.4b Match and/or sort factor pairs for numbers up to 50 with physical and/or visual representations (<i>for example, 50 matches: 1×50, 50×1, 10×5, 5×10, 2×25, and 25×2; 37 matches 37×1 and 1×37.</i>)	4.OA.4c Match factor pairs for whole numbers up to 20 with physical and/or visual representations (<i>for example, 12 matches: 1×12 and 12×1, 3×4 and 4×3, and 6×2 and 2×6.</i>)	<ul style="list-style-type: none"> Count the number of objects in an array. Relate a picture or objects to a number sentence. Recognize the symbols for multiplication (\times) and equals ($=$). Build groups of blocks into rows and columns (arrays). Identify the number of blocks in each row, column, and in total. Create multiple arrays using the same number of blocks. Recognize the orientation of an array can change. Recognize factors in an array are the same when two arrays are represented with different orientations. Match an array to its factors. Write a number sentence that represents a given array. Write a number sentence in more than one way. Engagement Statements (<i>demonstration of engaged in the topic</i>)

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings representing addition, subtraction, or multiplication word problems.
<i>Generate and analyze patterns.</i>				
4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. <i>For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</i>	4.OA.5a Given a rule for a pattern and its visual and/or physical representation, extend the pattern or identify or exclude objects or numbers that don't fit the rule of the pattern from physical and/or visual representations.	4.OA.5b Extend a shape or number pattern up to five terms given physical and/or visual representations.	4.OA.5c Extend a shape pattern two terms using a visual or physical representation (manipulatives).	<ul style="list-style-type: none"> Create a two-dimensional shape using triangles, rectangles, or squares when given physical objects such as pattern blocks. Create a two-dimensional shape using triangles, rectangles, or squares. Compose a larger two-dimensional shape from an original two-dimensional using triangles, rectangles, or squares when given physical objects such as pattern blocks. Match a given pattern composed triangles, rectangles, or squares. Identify the differences between an initial shape and its grown shape. <ul style="list-style-type: none"> Larger or smaller Colors of shapes Orientations Specific additions of shapes <p>Engagement Statements (<i>demonstration of engaged in the topic</i>)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
Numbers and Operations in Base Ten				
<i>Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.</i>				
4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right by applying concepts of place value, multiplication, or division.	4.NBT.1a Decompose multi-digit whole numbers by their place values and expanded form up to 100,000 with physical and/or visual representations (for example, 457: 4 hundreds, 5 tens, 7 ones; four hundred fifty-seven; $400 + 50 + 7$).	4.NBT.1b1 Given a whole number within the range of 1–999, decompose into place values of ones, tens and hundreds using physical and/or visual representations. AND 4.NBT.1b2 Given a whole number within the range of 1–999, identify the expanded form using physical and/or visual representations.	4.NBT.1c Given a whole number within the range of 1–99, identify the value in the ones place and/or the tens place using a place value chart or other visual/physical representation.	<ul style="list-style-type: none"> Count to 100 by ones and by tens. Recognize the numerals from 1 to 100. Represent numbers from 1 to 100 using physical objects. Know the word names for the numbers 1–100. Write numerals from 0 to 100. Explore place value tools. <ul style="list-style-type: none"> Base-10 blocks Place value chart 100's chart Cuisenaire rods Unifix cubes Distribute objects into groups of tens and ones. <ul style="list-style-type: none"> Compose and decompose numbers from 11 to 19 into a group of ten ones and some further ones by using objects. Understand that these numbers are composed of a group of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. Record the number of tens and ones in a group of objects or drawings. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones — called a “ten;” Understand the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Identify the location of the ones and tens on a place value chart. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings.
4.NBT.2 Read and write multi-digit whole numbers using standard form, word form, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.	4.NBT.2a1 Use place value knowledge to compare 2 numbers using $>$, $=$, and $<$ symbols along with physical and/or visual representations (whole numbers 1–10,000). 4.NBT.2a2 Read and write numbers up to 10,000 in standard and expanded form.	4.NBT.2b1 Use place value knowledge to compare 2 numbers using $>$, $<$, $=$ symbols along with physical and/or visual representations (whole numbers 1–1000). 4.NBT.2b2 Given a number in standard form or word form, write the number in expanded form. <i>For example, $206 = 200 + 6$ (whole numbers 1–1000).</i>	4.NBT.2c Match the word form or standard form of two-digit whole numbers with physical and/or visual representations of objects and place values. <i>For example, “25” or the word “twenty-five” is matched to a set of 25 objects and/or 2 tens and 5 ones cubes (whole numbers to 99).</i>	<ul style="list-style-type: none"> Recognize the numerals from 1 to 100. Represent numbers from 1 to 100 using physical objects. Know the word names for the numbers 1-100. Write numerals from 0 to 100. Explore place value tools. <ul style="list-style-type: none"> Base-10 blocks Place value chart 100's chart Cuisenaire rods Unifix cubes Distribute objects into groups of tens and ones. Record the number of tens and ones in a group of objects or drawings. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones — called a “ten;” Understand the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). <ul style="list-style-type: none"> Identify the location of the ones and tens on a place value chart. Recognize the standard form of a number when given the word name.

				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex ←————→ Least Complex				
				Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings.
4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place through 1,000,000.	4.NBT.3a Round whole numbers to any place using physical and/or linear visual representations (whole numbers to 10,000, number lines).	4.NBT.3b Round whole numbers to the nearest 10 or 100 using physical and/or linear visual representations (whole numbers to 1,000, number lines).	4.NBT.3c Round two-digit whole numbers to the nearest 10 using a physical and/or linear visual representation (whole numbers to 99, number lines).	<ul style="list-style-type: none"> Know what a number line is. Know the order of the numbers from 0 to 99. Identify a whole number on a number line marked with whole numbers up to 99. Identify 0 on a number line. Identify a missing whole number value on a number line marked with whole number up to 99. Compare distances of objects using a number line. Understand that 2 is the distance from 0 to 2 and 3 is the distance from 0 to 3 using standard units for all lengths from 1 to 99. Determine which tens a number comes between using a vertical or horizontal number line. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings (may include 100s chart) representing whole numbers up to 99.
<i>Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers less than or equal to 1,000,000.</i>				

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
4.NBT.4 Fluently add and subtract multi-digit whole numbers using a standard algorithm.	4.NBT.4a Add and subtract (with regrouping) 3-digit whole numbers using place value strategies and/or physical or visual representations (sums within 10,000).	4.NBT.4b Add and subtract up to two 3-digit whole numbers using place value strategies and/or physical or visual representations (including: adding two 2-digit whole numbers whose sums are less than 100 and may require regrouping; and adding two 3-digit numbers without regrouping whose sums are less than 1000; subtraction of two 2-digit or two 3-digit numbers without regrouping).	4.NBT.4c Add and subtract whole numbers using place value strategies and/or physical or visual representations. (Including sums of three one-digit whole numbers within 30, sums of 1-digit and 2-digit whole numbers with regrouping allowed in ones, and sums of two 2-digit whole numbers whose sums are within 100 without regrouping; subtraction of up to two 2-digit numbers without regrouping whose sums are within 100).	<ul style="list-style-type: none"> Represent a number with a set of physical objects or a drawing. Understand addition is the combining of two (or more) sets of objects. Understand subtraction is taking away of one amount of objects from another. Understand that addition and subtraction are opposites. Know the symbols for addition (+), subtraction (−), and equals (=). Add and subtract within 20 using strategies. Strategies may include: <ul style="list-style-type: none"> Counting on Making ten ($8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$) Decomposing a number leading to a ten ($13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$) Using the relationship between addition and subtraction; knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$ and Creating equivalent but easier or known sums (adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$). <p>Engagement Statements (<i>demonstration of engaged in the topic</i>)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings (may include 100s chart) representing whole numbers within 30.

				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex ←————→ Least Complex				
4.NBT.5 Multiply a whole number of up to four digits by a 1-digit whole number, and multiply two 2-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	4.NBT.5a Multiply a 2-digit whole number by a 2-digit whole number, using strategies based on place value and the properties of operations with arrays, area models, or other visual representations (products within 10,000).	4.NBT.5b Multiply a 3-digit whole number by a 1-digit whole number, using strategies based on place value and the properties of operations along with arrays, area models, or other physical representations (products within 1,000).	4.NBT.5c Multiply a multiple of 10 by a 1-digit whole number using arrays, area models, or other physical representations.	<ul style="list-style-type: none"> Create multiple groups of 10 using objects. Repeatedly add groups of 10 using physical objects. Count to 100 by 10s. Relate counting to addition by counting on 10 to add 10. Represent a number with a set of physical objects or a drawing. Record the number of tens and 0 ones in a group of objects or drawings for multiples of 10. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones — called a “ten”. Understand the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). <p>Engagement Statements (<i>demonstration of engaged in the topic</i>)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings (100s chart or multiplication chart).
4.NBT.6 Find whole-number quotients and remainders with up to 4-digit dividends and 1-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	4.NBT.6a Divide a 3-digit whole number by a 1-digit whole number using strategies based on place value, relationship between multiplication and division and the properties of operations using	4.NBT.6b Divide multiples of 10 up to 90 by a 1-digit whole number using strategies based on place value, the relationship between multiplication and division, and the properties of operations using	4.NBT.6c Determine whether a number is divisible by 2, 5, or 10 for numbers up to 50 using physical and/or visual representations (whole numbers to 100).	<ul style="list-style-type: none"> Identify groups of blocks 2s, 5s, and 10s. Share up to 50 objects equally between 2, 5, and 10 people (without remainders). <p>Engagement Statements (<i>demonstration of engaged in the topic</i>)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks).

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
	arrays, area models or other physical/visual representations (whole numbers answers with no remainders).	arrays, area models, or other physical/visual representations (whole numbers to 90 with no remainders).		
Numbers and Operations – Fractions				
<i>Extend understanding of fraction equivalence and ordering limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.</i>				
4.NF.1 Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	4.NF.1a Write or model equivalent fractions for denominators 2, 3, 4, 5, 6, 8, 10 when given rectangular fraction models or other physical/visual models. (fraction strips, number lines, area models).	4.NF.1b Identify equivalent fractions for denominators 2, 3, 4, 5, 6, 8, 10 when given rectangular fraction models or other physical visual models, <i>for example, matching model of $\frac{1}{2}$ and $\frac{2}{4}$.</i>	4.NF.1c Match fractions with their fraction model or other physical/visual models (limit to $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$).	<ul style="list-style-type: none"> Identify the same sized whole partitioned into 2, 3, and 4 equal shares. Describe the shares using the words halves, thirds, or fourths and quarters, and use the phrases half of, third of, or fourth of and quarter of. Describe the whole as two halves, three thirds, or four fourths. Recognize the value of a whole fractional shaded part. Identify $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$ when given a fraction model. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with area (rectangles) and length (number lines) fraction models.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←—————→ Least Complex				
4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.	4.NF.2a Compare two fractions using models and $>$, $=$, and $<$ symbols (limit denominators to 2, 3, 4, 5, 6, 8, 10).	4.NF.2b Using a given model, compare two fractions to identify which is “greater than”, “less than”, or “equal to” (limit fractions to unit fractions with denominators of 2, 3, 4, 5, 6, 8, 10).	4.NF.2c Determine which fraction is larger or smaller given pairs of fractions and their models (limit to $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$).	<ul style="list-style-type: none"> Directly compare two objects with a measurable attribute in common to see which object has “more of” or “less of” the attribute, and describe the difference. <ul style="list-style-type: none"> Directly compare the heights of two children, and describe one child as taller/shorter. Interact with physical objects and length models that represent fractional values. Visually compare fractional values using physical objects. <p>Engagement Statements (<i>demonstration of engaged in the topic</i>)</p> <ul style="list-style-type: none"> Interact with area (rectangles) and length (number lines) fraction models.
<i>Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified.)</i>				
4.NF.3 Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. <i>Examples:</i> $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2 \frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.	4.NF.3a Using physical and/or visual representation and fractions with denominators of 3, 4, 5, 6, 8, 10 and 100). a. Add and subtract fractions with like denominators. b. Decompose a mixed number into sums of fractions, for example,	4.NF.3b Using physical models or visual representations of fractions with denominators of 2, 3, 4, 5, 6, 8 and 10: a. Add and subtract fractions with like denominators. b. Decompose a mixed number into sums of unit fractions, for example, $1\frac{3}{4} = \frac{1}{4} +$	4.NF.3c Using physical models or visual representations and fractions with denominators of 2, 3 and 4 a. Add fractions with like denominators. b. Decompose a fraction into sums of unit fractions, for example, $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.	<ul style="list-style-type: none"> Identify the same sized whole partitioned into 2, 3, and 4 equal shares. Describe the whole as two halves, three thirds, or four fourths. Recognize the value of a whole fractional shaded part. Identify $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$ when given a fraction model. Partition rectangles into two, three, or four equal shares. Write a number sentence representing a whole partitioned into two, three, or four equal shares ($\frac{4}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$). Describe the shares using the words halves, thirds, or fourths and quarters, and use the

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←—————→ Least Complex				
<p>c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.</p> <p>d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.</p>	<p>$1\frac{3}{4} = \frac{4}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.</p> <p>c. Add and subtract fractions with like denominators <i>for example, $1\frac{1}{3} + 1\frac{1}{3} = 2\frac{2}{3}$ or $1\frac{2}{3} - \frac{1}{3} = 1\frac{1}{3}$</i> (including mixed numbers).</p> <p>d. Solve one-step real-world problems involving addition or subtraction of fractions with like denominators (referring to the same whole).</p>	<p>$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.</p> <p>c. Add and subtract a mixed number with a fraction of the same denominator.</p> <p>d. Solve real-world problems involving addition or subtraction of fractions with like denominators (referring to the same whole).</p>	<p>c. Know that one whole (partitioned into equal-sized parts) equals the sum of all its equal parts. <i>For example, 1 whole = 4 fourths ($\frac{4}{4}$) or 3 thirds ($\frac{3}{3}$).</i></p> <p>d. Solve one-step real-world problems involving addition of unit fractions (referring to the same whole).</p>	<p>phrases half of, third of, or fourth of and quarter of.</p> <ul style="list-style-type: none"> Describe the whole as two halves, three thirds, or four fourths in real-world contexts. Understand a fraction $\frac{1}{b}$ is the smallest fractional part of a whole. Count by unit fractions up to a whole. ($\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$) Use fraction models to combine equal sized shares with like denominators. Understand you can combine equal sized shares of fractional parts of the same whole like you can combine whole numbers. <p>Engagement Statements (<i>demonstration of engaged in the topic</i>)</p> <ul style="list-style-type: none"> Interact with area (rectangles) and length (number lines) fraction models.
<p>4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</p> <p>a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times (\frac{1}{4})$, recording the conclusion by the equation $\frac{5}{4} = 5 \times (\frac{1}{4})$ or $\frac{5}{4} = (\frac{1}{4}) + (\frac{1}{4}) + (\frac{1}{4}) + (\frac{1}{4})$.</p> <p>b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (\frac{2}{5})$ as $6 \times (\frac{1}{5})$,</p>	<p>4.NF.4a Solve real-world problems involving multiplying a fraction by a whole number up to 10 using visual fraction models. For example, three friends each ate $\frac{2}{8}$ of a pizza. How much pizza did they eat? (limit to fractions with denominators of 2, 3, 4, 6, and 8; no mixed numbers).</p>	<p>4.NF.4b Identify equivalent number sentences for fractions expressed as sums of unit fractions and products of a one-digit whole number multiplied by the same unit fraction, for example, $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ matches $3 \times \frac{1}{4}$ (limit to fractions with denominators of 2, 3, 4, 5, 6, 8 and 10; physical or visual</p>	<p>4.NF.4c Match a whole number with its given equivalent fraction using physical and/or visual representations for mathematical and word problems. (<i>For example, $4 = \frac{4}{1}$.</i>)</p>	<ul style="list-style-type: none"> Describe the whole as two halves, three thirds, or four fourths. Recognize the value of a whole fractional shaded part. Identify $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$ when given a fraction model. Partition rectangles into two, three, or four equal shares. Write a number sentence representing a whole partitioned into two, three, or four equal shares ($\frac{4}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$). Describe the shares using the words halves, thirds, or fourths and quarters, and use the phrases half of, third of, or fourth of and quarter of. Describe the whole as two halves, three thirds, or four fourths in real-world contexts.


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
recognizing this product as $\frac{6}{5}$. (In general, $n \times (\frac{a}{b}) = (\frac{n \times a}{b})$.) c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. <i>For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</i>		fraction models may be used).		Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with area (rectangles) and length (number lines) fraction models.
4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. <i>For example, express $\frac{3}{10}$ as $\frac{30}{100}$, and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$. In general, students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators, but addition and subtraction with unlike denominators is not a requirement at this grade.</i>	4.NF.5a Identify a fraction with a denominator of 10 as an equivalent fraction with denominator 100. <i>For example $\frac{30}{100} = \frac{3}{10}$.</i>	4.NF.5b Match a fraction with a denominator of 10 with its equivalent physical or visual model.	4.NF.5c Match a fraction with a denominator of 100 with its equivalent physical or visual model.	<ul style="list-style-type: none"> Understand content from 4.NF.1-4 prior to beginning instruction on 4.NF.5. Explore place values using place value models. (Base-10 blocks, Cuisenaire rods, pennies, dimes, etc.) Describe the whole as hundredths. Recognize the value of a fraction in hundredths using a model. Identify hundredths when given a fraction model. Partition rectangles into hundredths of equal shares. Describe the shares in terms of hundredths. Describe the shares in terms of hundredths in real-world contexts. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with place value models. (Base-10 blocks, Cuisenaire rods, pennies, dimes, etc.)

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
4.NF.6 Use decimal notation for fractions with denominators of 10 or 100. For example, rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.	4.NF.6a Rewrite a fraction with a denominator of 100 as a decimal using place value visual and/or physical representations. <i>For example, rewrite $\frac{62}{100}$ as 0.62.</i>	4.NF.6b Rewrite a fraction with a denominator of 10 as a decimal. For example, rewrite $\frac{2}{10}$ as 0.2 using place value, physical and/or visual representations.	4.NF.6c Match a collection of pennies or dimes to the visual model of the decimal. AND Select the decimal that represents a visual and/or physical model for a collection of pennies or dimes.	<ul style="list-style-type: none"> Understand content from 4.NF.5 prior to beginning instruction on 4.NF.6. Explore place values using place value models. (pennies and dimes) Recognize pennies and dimes. Know the names and values of pennies and dimes. Know the symbols for dollars (\$), cents (¢), and decimal point (.). Record the value of a collection of pennies using dollar or cent notation. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with pennies and dimes.
4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.	4.NF.7a Compare two decimals using place value models and the $<$, $>$, and $=$ symbols (limit to tenths with tenths and hundredths to hundredths, includes whole numbers to tens).	4.NF.7b Compare two decimals using place value models and the $<$, $>$, and $=$ symbols (limit to tenths with tenths and hundredths to hundredths, no whole numbers).	4.NF.7c Identify the tenths and hundredths place on a place value chart and in a given decimal using physical or visual representations.	<ul style="list-style-type: none"> Understand content from 4.NF.5-6 prior to beginning instruction on 4.NF.7. Explore place values using place value chart. Recognize the word names for tenths and hundredths. Know the symbol for decimal point (.). Understand the location of the decimal point on a place value chart. Recognize the value of a decimal in hundredths using a place value chart. Use language of <i>tenths</i> and <i>hundredths</i> in real-world contexts. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with place value chart.
Measurement and Data				
<i>Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</i>				

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
4.MD.1 Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. <i>For example, express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a two-column table of meters and centimeters includes the number pairs 1 and 100, 2 and 200, and 3 and 300.</i>	4.MD.1a Convert between km and m, m and cm, kg and g using place value charts or other physical/visual representations.	4.MD.1b Identify whether a measurement is “more than,” “less than,” or “same as” another metric measurement using a place value chart or other physical/visual models. <i>For example, 1.5 kg is larger than 500 g.</i>	4.MD.1c Determine the best metric unit to measure a specific real-world item (metric units include centimeters and meters, kilograms and grams, liters).	<ul style="list-style-type: none"> Describe measurable attributes of a single object using terms such as heavy/light, long/short, short/tall, etc. Explore measurements in cups and liters; inches, meters, and centimeters; grams and kilograms. Directly compare two amounts of liquid using measuring cups and liters to see which one holds more and which one holds less. Directly compare two beakers in liters to see which one holds more and which one holds less. Directly compare two distance measurements in centimeters, inches, and meters using rulers and meter sticks. Directly compare two measurements in grams and kilograms using different types of scales. Use different types of scales to measure mass of objects in kilograms and grams. Use different types of containers to measure volume of liquids in liters. Use different types of meter sticks and rulers to measure distance in meters and centimeters. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with measurement tools for length, volume, and mass.
4.MD.2 Solve real-world problems involving money, time, and metric measurement. a. Using models, add and subtract money and express the answer in decimal notation. b. Using number line diagrams, clocks, or other models, add and	4.MD.2a1 Solve real-world problems involving addition or subtraction of coins and bills using visual and/or physical representations	4.MD.2b1 Solve real-world problems with addition of collections of coins or bills with visual and/or physical representations (limit amounts to less than \$50).	4.MD.2c1 Identify the value of all coins. Find the total of a collection of all pennies or all dimes or all nickels. AND	<ul style="list-style-type: none"> Understand content from 4.NF.5-7 before beginning instruction with pennies, nickels, and dimes. Understand content from 4.MD.2 before beginning instruction on 4.MD.3c. Explore place values using place value models. (pennies, nickels, and dimes) Recognize pennies, nickels, and dimes.


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
subtract intervals of time in hours and minutes. c. Add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects.	(limit amounts to less than \$100). AND 4.MD.2a2 Solve word problems involving addition and subtraction of time intervals in 15 minutes with visual and/or physical representations. AND 4.MD.2a3 Solve real-world problems involving mass or volume by selecting appropriate operations with physical and/or visual representations.	AND 4.MD.2b2 Solve word problems involving addition of time intervals of 30 minutes with visual and/or physical representations. AND 4.MD.2b3 Solve real-world problems by measuring liquid volumes and masses of objects using standard units of measure with physical and/or visual representations.	4.MD.2c2 Solve word problems involving addition of time intervals of one hour with visual and/or physical representation. AND 4.MD.2c3 Solve real-world problems by selecting the appropriate tool to measure metric volume or mass with visual and physical representations.	<ul style="list-style-type: none"> Describe differences between U.S. coins. Know the names and values of pennies, nickels, and dimes. Know the symbols for dollars (\$), cents (¢), and decimal point (.). Record the value of a collection of pennies, nickels, or dimes using dollar or cent notation. Relate counting to 12 to telling time on an analog clock. Tell time using a digital clock. Read the hour hand on an analog clock at different times of a day. Know the meaning of the hour and the minute hands on an analog clock. Use different types of scales to measure mass of objects in kilograms and grams. Use different types of containers to measure volume of liquids in liters. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with pennies and dimes. Interact with analog clocks. Interact with measurement tools for volume and mass.
4.MD.3 Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. <i>For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.</i>	4.MD.3a1 Given a rectangle with side lengths marked, find area (limit to whole number measurements). 4.MD.3a2 Solve real-world problems involving perimeter. Find the perimeter	4.MD.3b1 Find the area of rectangles by counting unit squares and understand that a square with a side length of 1 unit is called a “unit square.”	4.MD.3c1 Find the area of rectangles with whole-number side lengths by counting unit squares. 4.MD.3c2 Find the perimeter of rectangles by counting the	<ul style="list-style-type: none"> Identify surfaces where an area can be measured. Lay unit squares on a flat surface. Count unit squares on a flat surface. Arrange unit squares into rows and columns. Identify surfaces where a perimeter can be measured. Lay inch squares around a flat surface.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
	or given the perimeter, find the missing side length (limit to whole numbers).	4.MD.3b2 Find the perimeter of rectangles drawn on grid paper (whole numbers only).	number of unit squares that fit around the shape.	<ul style="list-style-type: none"> Count inch squares all the way around a flat surface. Measure length and width with units and record the measurement. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with flat two-dimensional surfaces.
<i>Represent and interpret data.</i>				
4.MD.4 Display and interpret data in graphs (picture graphs, bar graphs, and line plots) to solve problems using numbers and operations for this grade.	4.MD.4a Interpret data from a given line, picture, or bar graph to solve a multi-step problem (limit to whole numbers).	4.MD.4b Interpret data represented in a graph by solving one-step “how many more” and “how many less” problems (limit to whole numbers).	4.MD.4c Given a bar or picture graph, build a graph based on student sorted data. For example, votes for 4 different candidates or weather types, or occurrences of event or behavior.	<ul style="list-style-type: none"> Classify objects into categories. Count the number of objects in each category. Create a bar graph or picture graph with a scale of 1 by stacking physical objects. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with a bar graph or picture graph.
<i>Geometric measurement: Understand concepts of angle and measure angles.</i>				
4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. a. Understand an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc	4.MD.5a Label the parts of an angle (two rays, common endpoint) and identify, 45-, 90-, 180-, and 360-degree angles.	4.MD.5b Match words to the parts of an angle (two rays, common endpoint).	4.MD.5c Identify an angle in a given shape.	<ul style="list-style-type: none"> Explore angle measurement using a protractor or angle ruler. Recognize angles. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with a protractor or angle ruler.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
<p>between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a “one-degree angle,” and can be used to measure angles.</p> <p>b. Understand an angle that turns through n one-degree angles is said to have an angle measure of n degrees.</p>				
<p>4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</p>	<p>4.MD.6a Measure 45-, 90-, and 180-degree angles.</p>	<p>4.MD.6b Match measures and diagrams of 45-, 90-, and 180-degree angles.</p>	<p>4.MD.6c Given two measuring tools, select a protractor as a tool to measure and sketch angles.</p>	<ul style="list-style-type: none"> Explore angle measurement using a protractor or angle ruler. Recognize angles. <p>Engagement Statements (<i>demonstration of engaged in the topic</i>)</p> <ul style="list-style-type: none"> Interact with a protractor or angle ruler.
<p>4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</p>	<p>4.MD.7a Solve measurement problems involving angles using addition and subtraction.</p>	<p>4.MD.7b Solve measurement problems involving angles using addition.</p>	<p>4.MD.7c Solve addition measurement problems involving angles to show that two smaller angles make a larger angle.</p>	<ul style="list-style-type: none"> Understand content from 4.MD.5-6 and 4.NBT.4 before beginning instruction 4.MD.7 Explore angle measurement using an angle ruler. Recognize angles. Use two angle rulers to combine angles. Create a larger angle by combining two smaller angles. Find sums of two 2-digit whole numbers angle measures whose sums are within 90 without regrouping. <p>Engagement Statements (<i>demonstration of engaged in the topic</i>)</p> <ul style="list-style-type: none"> Interact with an angle ruler.
Geometry				
<i>Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</i>				

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
4.G.1 Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	4.G.1a Match word to corresponding picture of perpendicular and parallel lines and angles (right, acute, obtuse).	4.G.1b Match word to corresponding picture of points, lines, line segments rays, and angles.	4.G.1c Identify a point, line, and a line segment.	Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with two-dimensional figures.
4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.	4.G.2a Label a two-dimensional shape to show understanding of parallel or perpendicular lines.	4.G.2b Sort two-dimensional shapes based on presence of parallel and/or perpendicular lines.	4.G.2c Match two-dimensional shapes in the environment based on parallel lines.	<ul style="list-style-type: none"> Identify shapes as two-dimensional (lying in a plane, flat) or three-dimensional (solid). Name shapes regardless of their orientations or overall size. Recognize lines. Understand parallel lines never intersect. Engagement Statements (<i>demonstration of engaged in the topic</i>) <ul style="list-style-type: none"> Interact with two- and three-dimensional objects in their environment.

Grade 5

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
Operations and Algebraic Thinking				
Write and interpret numerical expressions.				
5.OA.1 Use parentheses in numerical expressions and evaluate expressions with this symbol. Formal use of algebraic order of operations is not necessary.	5.OA.1a Solve a two-step expression involving addition and subtraction and parentheses. <i>For example, $5 + (6 - 3) = 5 + 3 = 8$ (limit to one-digit whole numbers).</i>	5.OA.1a Identify the first step in solving a two-step problem involving addition and subtraction.	5.OA.1c Identify parentheses as a marker of a group in a number sentence.	<ul style="list-style-type: none"> Identify a number sentence. Recognize the symbols for addition (+), subtraction (−), multiplication (×), division (÷), and equals (=). Read and interpret a traditional one-step number sentence ($2 \times 3 = \chi$). Relate a picture or objects to a number sentence. Know that a symbol χ can represent a missing value. Know the parentheses are evaluated first within an expression. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with linear models or physical objects (blocks) or drawings representing addition, subtraction, multiplication, or division.
5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. <i>For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$.</i>	5.OA.2a Write a two-step numerical expression when given a number sentence; do not calculate.	5.OA.2b Match a two-step number sentence with its expression. <i>For example, add 2 and 2 then subtract</i>	5.OA.2c Match a one-step number sentences with its expression/ <i>For example, 2 plus 2 = 2+2.</i>	<ul style="list-style-type: none"> Identify a number sentence. Recognize the symbols for addition (+), subtraction (−), multiplication (×), and equals (=). Read and interpret a traditional one-step number sentence in a context ($2 \times 3 = \chi$).


				Learning Progression <i>Building the Base & Engagement</i>										
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Most Complex ←————→ Least Complex														
<i>Recognize that $3 \times (18,932 + 921)$ is three times as large as $18,932 + 921$, without having to calculate the indicated sum or product.</i>		<i>1 matches $(2+2) - 1$.</i>		<ul style="list-style-type: none">Relate a picture or objects to a number sentence.Know that a symbol χ can represent a missing value.Know the parentheses are evaluated first within an expression. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none">Interact with linear models or physical objects (blocks) or drawings representing addition, subtraction, or multiplication.										
<i>Analyze patterns and relationships.</i>														
<p>5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. <i>For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other</i></p>	<p>5.OA.3a Given a rule for a numerical pattern and a two-column table missing several terms, complete the table and graph in the first quadrant.</p> <p>Add 4</p> <table><tr><td>1</td><td>5</td></tr><tr><td>2</td><td>6</td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td>8</td></tr><tr><td>5</td><td></td></tr></table>	1	5	2	6	3		4	8	5		<p>5.OA.3b Plot 2 ordered pairs in the first quadrant of a coordinate grid.</p>	<p>5.OA.3c Given a coordinate grid marked with locations of familiar locations (school, home, library, grocery), identify the location for an ordered pair.</p>	<ul style="list-style-type: none">Identify points on a horizontal number line (scale limited to whole numbers 1-10).Identify points on a vertical number line (scale limited to whole numbers 1-10).Understand a coordinate grid is formed by a vertical and horizontal number line. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none">Interact with coordinate grid.
1	5													
2	6													
3														
4	8													
5														


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
sequence. Explain informally why this is so.				
Numbers and Operations in Base Ten				
<i>Understand the place value system.</i>				
5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.	5.NBT.1a Decompose multi-digit whole numbers by their place values and expanded form to show understanding that a digit in one place represents 10 times as much as it represents in the place to its right.	5.NBT.1b Decompose multi-digit whole numbers by their place values and expanded form up to 100,000 with physical and/or visual representations. <i>For example, 457: 4 hundreds, 5 tens, 7 ones; four hundred fifty-seven; $400 + 50 + 7$.</i>	5.NBT.1c1 Given a whole number within the range of 1–999, decompose into place values of ones, tens, and hundreds using physical and/or visual representation. AND 5.NBT.1c2 Given a whole number within the range of 1–999, identify the expanded form using physical	<ul style="list-style-type: none"> Count to 100 by ones and by tens. Count to 1,000 by hundreds and by tens. Recognize the numerals from 1 up to 1,000. Represent numbers from 1 to 1,000 using physical objects or technology. Know the word names for the numbers 1–1,000. Write numerals from 0 to 1,000. Explore place value tools including with technology. <ul style="list-style-type: none"> Base-10 blocks Place value chart 100's chart Cuisenaire rods Unifix cubes

				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex ←————→ Least Complex				
			and/or visual representations.	<ul style="list-style-type: none"> Distribute objects into groups of hundreds, tens, and ones. <ul style="list-style-type: none"> Compose and decompose numbers from 11 to 19 into a group of ten ones and some further ones by using objects. Understand that these numbers are composed of a group of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. Record the number of tens and ones in a group of objects or drawings. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones — called a “ten;” Understand the following as special cases: 100 can be thought of as a bundle of ten tens — called a “hundred” Understand the following special cases: 1,000 can be thought of as a bundle of ten hundreds — called a “thousand” Understand the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). Understand the numbers 100, 200, 300, 400, 500, 600, 700, 800, and 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 ones).

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←				→ Least Complex
				<ul style="list-style-type: none"> Identify the location of the ones, tens, and hundreds on a place value chart. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings.
5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	5.NBT.2a Identify the product of a two-digit decimal and a power of 10 (limit decimals to tenths and powers of 10 to 10^3).	5.NBT.2b Identify the power of 10 written as a base and an exponent for its multiplication sentence. <i>For example, $10^2 = 10$; $10^2 = 10 \times 10$; $10^2 = 10 \times 10 \times 10$.</i> Identify the product of a 2-digit whole number and a power of 10 (limit powers of 10 to 1,000 or 10^3).	5.NBT.2c Identify the number of zeroes in a product of a one-digit whole number and 10 or 100 using place value strategies or physical representations.	<ul style="list-style-type: none"> Content works in conjunction with 5.NBT.5. Create multiple groups of 10 using objects. Repeatedly add groups of 10 using physical objects. Relate counting to addition by counting on 10 to add 10. Understand the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). Know the symbols for multiplication (\times) and equals ($=$). Relate multiplication to repeated addition by writing a number sentence. Write a number sentence to represent a situation involving a multiple of 10. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings (100s chart or multiplication chart).

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
<p>5.NBT.3 Read, write, and compare decimals to thousandths.</p> <p>a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.</p> <p>b. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p>	<p>5.NBT.3a Compare two decimal numerals written to the hundredths place using $>$, $=$ and $<$ symbols with visual and/or physical representations.</p>	<p>5.NBT.3b Compare two decimal models to the tenths place using $>$, $=$ and $<$ symbols with visual and/or physical representations.</p>	<p>5.NBT.3c Match visual or physical representations or models of tenths and determine which is “more than”, “same as”, or “Less than”.</p>	<ul style="list-style-type: none"> Content works in conjunction with 5.NBT.4 and 5.NBT.7. Recognize pennies and dimes. Know the names and values of pennies and dimes. Know the symbols for dollars (\$), cents (¢), and decimal point (.). Record the value of a collection of pennies using dollar and cent notation. Explore place values using place value models. (pennies and dimes) Recognize the word names for tenths and hundredths. Understand the location of the decimal point on a place value chart. Recognize the value of a decimal in hundredths using a place value chart. Represent the value of a collection of pennies and dimes on a place value chart. Use language of <i>tenths</i> and <i>hundredths</i> in real-world contexts. Orally identify (without using inequality symbols) whether the number of objects in one group is greater/more than, less/fewer than, or the same as the number of objects in another group, not to exceed 10 objects in each group. Compare (without using inequality symbols) two numbers between 0 and 10 when presented as written numerals. Directly compare two objects with a measurable attribute in common to see which object has “more of” or “less of” the attribute,


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
				<p>and describe the difference. For example, directly compare the heights of two children, and describe one child as taller/shorter.</p> <ul style="list-style-type: none"> Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with place value models. (pennies, dimes, place value charts, etc.)
5.NBT.4 Use place value understanding to round decimals to any place, millions through hundredths.	5.NBT.4a Round decimals to a given place using a number line or other physical/visual representation (limit rounding to the nearest tenth, one, ten or hundred).	5.NBT.4b Round decimals with a value in the tenths place to the nearest whole number using a number line or other physical/visual representation.	5.NBT.4c Identify whether a decimal is closer to 0 or 1 using physical and/or visual representations, including money (limit decimal place values to hundredths).	<ul style="list-style-type: none"> Content works in conjunction with 5.NBT.3 and 5.NBT.7. Know what a number line is. Know the order of the numbers from 0 up to 99. Identify a whole number on a vertical or a horizontal number line marked with whole numbers up to 99 (scale limited to whole numbers 1 or 10).

				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex  Least Complex				
				<ul style="list-style-type: none"> Identify 0 on a number line. Identify a missing whole number value on a number line marked with whole numbers up to 99. Compare distances of objects using a number line. Understand that 2 is the distance from 0 to 2 and 3 is the distance from 0 to 3 using standard units for all lengths from 1 to 99. Determine which tens a group of pennies comes between using a vertical or horizontal number line. Determine which tenths a group of pennies and dimes comes between using a vertical or horizontal number line. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings (may include 100s chart) representing whole numbers up to 99.
<i>Perform operations with multi-digit whole numbers and with decimals to hundredths.</i>				
5.NBT.5 Fluently multiply multi-digit whole numbers using a standard algorithm.	5.NBT.5a Multiply a 3-digit whole number by a 2-digit whole number with visual and/or physical representations.	5.NBT.5b Multiply 2-digit by 2-digit whole numbers with physical and/or visual representations	5.NBT.5c Multiply two 2-digit whole numbers, where one factor is a multiple of 10, using models with physical or/and	<ul style="list-style-type: none"> Content works in conjunction with 5.NBT.2. Count to 10. Count to 10 using objects. Count to 100 by 10s. Create multiple groups of 10 using objects. Repeatedly add groups of 10 using physical objects.

				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex ←————→ Least Complex				
			visual representations	<ul style="list-style-type: none"> Relate counting to addition by counting on 10 to add 10. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones — called a “ten”. Understand the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). Know the symbols for multiplication (\times) and equals ($=$). Relate multiplication to repeated addition by writing a number sentence. Represent a number with a set of physical objects or a drawing. Record the number of tens and 0 ones in a group of objects or drawings for multiples of 10. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings (100s chart or multiplication chart).
5.NBT.6 Find whole-number quotients of whole numbers with up to 4-digit dividends and 2-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship	5.NBT.6a Divide whole numbers using strategies based on place value, the relationship between	5.NBT.6b Divide whole numbers using strategies based on place value, the relationship	5.NBT.6c Divide a 2-digit whole number by a 1-digit whole number using strategies based on place value, the	<ul style="list-style-type: none"> Identify 2, 3, 4, 5, 6, and 10 blocks. Identify groups of blocks 2s, 3a, 4s, 5s, 6s, and 10s. Share up to 10, 20, 30, 40, 50 objects equally between 2, 3, 4, 5, 6, and 10 people (without remainders).

				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex ←————→ Least Complex				
between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	multiplication and division, the properties of operations and physical and/or visual representations (limit to 4-digit whole number by a 1-digit whole number, no remainders).	between multiplication and division, the properties of operations and physical and/or visual representations (3-digit by 1-digit whole numbers with no zeroes in the tens place, no remainders).	relationship between multiplication and division, the properties of operations and using physical and/or visual representations (products limited to 100, no remainders).	<ul style="list-style-type: none"> Build groups of blocks into rows and columns (arrays). Count the number of blocks in a given array. Identify the number of blocks in each row and each column. Recognize factors in an array. Relate a picture or objects to a number sentence. Identify a number sentence. Recognize the symbols for multiplication (\times), division (\div), and equals ($=$). Read and interpret a traditional one-step number sentence ($6 \div 3 = \square$). Know that a symbol \square can represent a missing value. Recognize related multiplication and division number sentences. Write the number sentence to express the total as a product of two factors. Solve 1-step number sentences involving multiplication or division. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings representing multiplication and division.
5.NBT.7 Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models or	5.NBT.7a1 Add and subtract decimals to hundredths with physical and/or	5.NBT.7b1 Add and subtract decimals to hundredths using	5.NBT.7c1 Add and subtract decimals to tenths using physical and/or	<ul style="list-style-type: none"> Content works in conjunction with 5.NBT.3 and 5.NBT.4.

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
<div> <div>Most Complex</div> <div>←</div> <div>→</div> <div>Least Complex</div> </div>				
<p>drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, or multiplication and division; relate the strategy to a written method and explain the reasoning used.</p> <p>a. Add and subtract decimals, including decimals with whole numbers (whole numbers through the hundreds place and decimals through the hundredths place).</p> <p>b. Multiply whole numbers by decimals (whole numbers through the hundreds place and decimals through the hundredths place).</p> <p>c. Divide whole numbers by decimals and decimals by whole numbers (whole numbers through the tens place and decimals less than one through the hundredths place, using numbers whose division can be readily modeled). <i>For example, 0.75 divided by 5, 18 divided by 0.6, or 0.9 divided by 3.</i></p>	<p>visual representations with word problems.</p> <p>AND</p> <p>5.NBT.7a2 Multiply and divide up to a 3-digit whole number by a 2-digit number with a digit in the tenths place value, with physical and/or visual representations, with or without word problem. <i>For example, 312×1.2.</i></p>	<p>physical and/or visual representations with and without word problems.</p> <p>AND</p> <p>5.NBT.7b2 Multiply or divide up to a 3-digit whole number by a decimal to the tenths place with physical and/or visual representations, with or without word problem. <i>For example, 312×0.3.</i></p>	<p>visual representations with word problems related to money.</p> <p>AND</p> <p>5.NBT.7c2 Multiply a decimal to the tenths place by a single digit whole number with physical and/or visual representations, with or without word problem. <i>For example, 7×0.1.</i></p>	<ul style="list-style-type: none"> Explore place values using place value models. (Base-10 blocks, Cuisenaire rods, pennies, nickels, dimes, quarters, etc.) Know the names and values of pennies, nickels, dimes, and quarters. Know the symbols for dollars (\$), cents (¢), and decimal point (.) Match a collection of pennies, nickels, dimes, or quarters to the visual model of the decimal. Record the value of a collection of pennies, nickels, or dimes using dollar or cent notation. Represent a value of a collection of coins on a place value chart. Recognize the value of a decimal to hundredths using a place value chart. Use language of <i>tenths</i> and <i>hundredths</i> in real-world contexts. Add and subtract collections of coins (pennies, nickels, dimes, quarters). Write the number sentence for the addition or subtraction of two collections of coins. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with linear models or place value models. (Base-10 blocks, Cuisenaire rods, pennies, dimes, etc.)
Numbers and Operations – Fractions				

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
Use equivalent fractions as a strategy to add and subtract fractions (fractions need not be simplified.).				
5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. <i>For example, use visual models and properties of operations to show $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$.</i>	5.NF.1a Add and subtract fractions with fraction models or other physical and/or visual representations. <i>For example, $\frac{2}{4} + \frac{2}{8} = \frac{6}{8}$. Limit denominators to multiple pairs of 3 and 6, 2 and 4, 4 and 8, 5 and 10, or 10 and 100, may include mixed numbers.</i>	5.NF.1b Add and subtract fractions with same denominator fraction models or other physical and/or visual representations (limit denominators to 2, 3, 4, 5, 6, 8, and 10).	5.NF.1c Add and subtract fractions with the same denominator using fraction models or other physical and/or visual representations (limit denominators to 2, 3, 4, and 10).	<ul style="list-style-type: none"> Content works in conjunction with 5.NF.2. Identify the same sized whole partitioned into 2, 3, 4, and 10 equal shares. Describe the whole as two halves, three thirds, four fourths, or ten tenths. Recognize the value of a whole fractional shaded part. Identify $\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$ when given a fraction model. Partition rectangles into two, three, four, or ten equal shares. Write a number sentence representing a whole partitioned into two, three, or four equal shares ($\frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$). Describe the shares using the words halves, thirds, fourths and quarters, or tenths and use the phrases half of, third of, fourth of and quarter of or tenth of. Describe the whole as two halves, three thirds, four fourths or ten tenths in real-world contexts. Understand a fraction $\frac{1}{6}$ is the smallest fractional part of a whole. Count by unit fractions up to a whole. ($\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$) Use fraction models to combine equal sized shares with like denominators. Understand you can combine equal sized shares of fractional parts of the same whole like you can combine whole numbers.


				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Match fractions with their fraction model or other physical/visual models (limit to denominators of 2, 3, 4, and 10). <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with area (rectangles) and length (number lines) fraction models.
5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. <i>For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$.</i>	5.NF.2a Solve addition and subtraction fraction word problems with fraction models or other physical and/or visual representations. Limit denominators to multiple pairs of 3 and 6, 2 and 4, 4 and 8, 5 and 10, or 10 and 100, may include mixed numbers.	5.NF.2b Solve addition and subtraction word problems involving fractions with like denominators with fraction models or other physical and/or visual representations. Limit denominators to 2, 3, 4, 5, 6, 8 and 10.	5.NF.2c Solve addition and subtraction word problems involving fractions with like denominators with fraction models or other physical and/or visual representations. Limit denominators to 2, 3, 4, and 10.	<ul style="list-style-type: none"> Content works in conjunction with 5.NF.1. Identify the same sized whole partitioned into 2, 3, 4, and 10 equal shares. Describe the whole as two halves, three thirds, four fourths, or ten tenths. Recognize the value of a whole fractional shaded part. Identify $\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$ when given a fraction model. Partition rectangles into two, three, four, or ten equal shares. Write a number sentence representing a whole partitioned into two, three, or four equal shares ($\frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$). Describe the shares using the words halves, thirds, fourths and quarters, or tenths and use the phrases half of, third of, fourth of and quarter of or tenth of. Describe the whole as two halves, three thirds, four fourths or ten tenths in real-world contexts. Understand a fraction $\frac{1}{n}$ is the smallest fractional part of a whole.

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←				→ Least Complex
				<ul style="list-style-type: none"> Count by unit fractions up to a whole. ($\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$) Use fraction models to combine equal sized shares with like denominators. Understand you can combine equal sized shares of fractional parts of the same whole like you can combine whole numbers. Match fractions with their fraction model or other physical/visual models (limit to denominators of 2, 3, 4, and 10). <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with area (rectangles) and length (number lines) fraction models.
Apply and extend previous understandings of multiplication and division to multiply and divide fractions (fractions need not be simplified)				
5.NF.3 Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. <i>For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people, each person has a share</i>	5.NF.3a Given fraction models or other physical/visual representations with or without a familiar context and limited to whole numbers up to 50, identify the number sentence when represented as a fraction. For example, 48 pounds of nuts are shared	5.NF.3b Given fraction models or other physical/visual representations with or without a familiar context and limited to whole numbers up to 20 with no remainders, identify the number sentence when represented as a fraction. <i>For</i>	5.NF.3c Match a fraction to its division problem shown with a fraction model or other physical or visual representation (limit fractions to denominators of 2, 3, 4, and 10).	<ul style="list-style-type: none"> Understand content from 5.NF.1 prior to beginning instruction on 5.NF.3. Recognize the symbols for division (\div) and equals ($=$). A fraction represents division ($3 \div 4 =$ $\frac{3}{4}$). <p>Understand that 34 can be read as 3 divided by 4.</p> <p>Engagement Statements (demonstration of engaged in the topic)</p>


				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←—————→ Least Complex				
<i>of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</i>	by 6 children and $\frac{48}{6}$. Solve the number sentence from above. For example, 48 pounds of nuts shared by 6 children equals 8 (pounds of nuts for each child).	<i>example, 4 children share 3 candy bars matches (3 candy bars divided by 4) $\frac{3}{4}$, or 12 pounds of nuts shared by 6 children equals $\frac{12}{6}$.</i>		<ul style="list-style-type: none"> Interact with area (rectangles) and length (number lines) fraction models.
5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(\frac{a}{b}) \times q$ as a part of a partition of q into b equal parts, equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(\frac{2}{3}) \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $(\frac{2}{3}) \times (\frac{4}{5}) = \frac{8}{15}$. (In general, $(\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}$.) b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and	5.NF.4a Solve real-world problems involving multiplying a fraction by a whole number up to 10 using visual fraction models. For example, <i>3 friends each ate $\frac{2}{8}$ of a pizza. How much pizza did they eat?</i> (Limit to fractions with denominators of 2, 3, 4, 6, and 8; may involve mixed numbers.)	5.NF.4b Solve real-world problems involving multiplying a fraction by a whole number up to 10 using visual fraction models. For example, <i>3 friends each ate $\frac{2}{8}$ of a pizza. How much pizza did they eat?</i> (Limit to fractions with denominators of 2, 3, 4, 6, and 8; no mixed numbers.)	5.NF.4c Identify equivalent number sentences for fractions expressed as sums of unit fractions and products of a 1-digit whole number multiplied by the same unit fraction. For example, $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ matches $3 \times \frac{1}{4}$. (Limit to fractions with denominators of 2, 3, 4, 5, 6, 8 and 10; physical or visual fraction models may be used.)	<ul style="list-style-type: none"> Understand content from 5.NF.3 prior to beginning instruction on 5.NF.4. Identify the same sized whole partitioned into 2, 3, 4, 5, 6, 8, and 10 equal shares. Describe the whole as two halves, three thirds, four fourths, etc. Recognize the value of a whole fractional shaded part. Identify $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc. when given a fraction model. Partition rectangles into 2, 3, 4, 5, 6, 8, or 10 equal shares. Write a number sentence representing a whole partitioned into 2, 3, 4, 5, 6, 8, or 10 equal shares ($\frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$). Describe the shares using the words halves, thirds, or fourths and quarters, etc. use the phrases half of, third of, or fourth of and quarter of, etc. Describe the whole as two halves, three thirds, four fourths, etc. in real-world contexts. Understand a fraction $\frac{1}{b}$ is the smallest fractional part of a whole.


				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
represent fraction products as rectangular areas.				<ul style="list-style-type: none"> Count by unit fractions up to a whole. ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc.) Use fraction models to combine equal sized shares with like denominators. Understand you can combine equal sized shares of fractional parts of the same whole like you can combine whole numbers. Draw or create a picture that represents <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with area (rectangles) and length (number lines) fraction models.
5.NF.5 Interpret multiplication as scaling (resizing). a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence	5.NF.5a Match a multiplication problem involving a fraction multiplied by a whole number to its represented picture to show knowledge that product will be smaller than whole number in problem with visual and/or physical representations (limit to fractions with denominators of 2, 3, 4, 6, and 8; may involve mixed	5.NF.5b Match a multiplication problem involving a fraction multiplied by a whole number to its represented picture to show knowledge that product will be smaller than whole number in problem with visual and/or physical representations (limit to fractions with denominators of 2, 3, 4, 6, and 8; no mixed	5.NF.5c Match a multiplication problem using $\frac{1}{2}$ multiplied by a whole number to its represented picture to show knowledge that product will be smaller than whole number in problem with visual and/or physical representations. <i>For example, $3 \times \frac{1}{2} = 1\frac{1}{2}$.</i> Compare the product to the factors.	<ul style="list-style-type: none"> Understand content from 5.NF.3-4 prior to beginning instruction on 5.NF.5. Identify the same sized whole partitioned into 2 equal shares. Describe the whole as two halves. Recognize the value of a whole fractional shaded part. Partition rectangles into 2 equal shares. Identify $\frac{1}{2}$ or $\frac{2}{2}$ when given a fraction model. Count by unit fractions of $\frac{1}{2}$ up to a whole and onto more than one whole. Write a number sentence representing more than one whole partitioned into 2 equal shares ($\frac{4}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$). Describe the shares using the words halves use the phrase half of.

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
$\frac{a}{b} = \frac{(n \times a)}{(n \times b)}$ to the effect of multiplying $\frac{a}{b}$ by 1.	numbers.) Compare the product to the factors using <, >, = symbols.	numbers). Compare the product to the factors using <, >, = symbols.		<ul style="list-style-type: none"> Describe the whole as two halves, three halves, four halves, etc. in real-world contexts. Understand a unit fraction $\frac{1}{b}$ is the smallest fractional part of a whole. Use fraction models to combine equal sized shares with like denominators. Understand you can combine equal sized shares of fractional parts of the same whole like you can combine whole numbers. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with area (rectangles) and length (number lines) fraction models.
5.NF.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	5.NF.6a Using physical models or visual representations of fractions with denominators of 2, 3, 4, 5, 6, 8 and 10, solve real-world problems involving multiplication of fractions and mixed numbers.	5.NF.6b Using physical models or visual representations of fractions with denominators of 2, 3, 4, 5, 6, 8 and 10, solve real-world problems involving multiplication of fractions and whole numbers.	5.NF.6c Match the real-world word problem to the number sentence that involves multiplying a fraction by a whole number with physical and/or visual representation.	<ul style="list-style-type: none"> Understand content from 5.NF.3-5 prior to beginning instruction on 5.NF.5. Identify the same sized whole partitioned into 2 equal shares. Describe the whole as two halves. Recognize the value of a whole fractional shaded part. Partition rectangles into 2 equal shares. Identify $\frac{1}{2}$ or $\frac{2}{2}$ when given a fraction model. Count by unit fractions of $\frac{1}{2}$ up to a whole and onto more than one whole. Write a number sentence representing more than one whole partitioned into 2 equal shares ($\frac{4}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$). Describe the shares using the words halves use the phrase half of.

				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex  Least Complex				
				<ul style="list-style-type: none"> Describe the whole as two halves, three halves, four halves, etc. in real-world contexts. Understand a unit fraction $\frac{1}{b}$ is the smallest fractional part of a whole. Use fraction models to combine equal sized shares with like denominators. Understand you can combine equal sized shares of fractional parts of the same whole like you can combine whole numbers. Know the symbols for multiplication (\times) and equals ($=$). Relate multiplication to repeated addition by writing an equivalent number sentence. Represent a number with a set of physical objects or a drawing. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with area (rectangles) and length (number lines) fraction models.
5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. In general, students able to multiply fractions can develop strategies to divide fractions, by reasoning about the relationship between multiplication and division, but	5.NF.7a Solve real-world problems involving division of a fraction by a whole number using visual and/or physical representations (limit to denominators of 2,	5.NF.7b Given a real-world context and its equation involving division of a fraction by a whole number, find the missing factor (limit numbers to 5 or less).	5.NF.7c Match the real-world word problem and its number sentence involving division of a fraction by a whole number to the physical and/or visual representation.	<ul style="list-style-type: none"> Understand content from 5.NF.3-6 prior to beginning instruction on 5.NF.5. Identify the same sized whole partitioned into 2 equal shares. Describe the whole as two halves. Recognize the value of a whole fractional shaded part. Partition rectangles into 2 equal shares. Identify $\frac{1}{2}$ or $\frac{2}{2}$ when given a fraction model.


				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
<div>Most Complex</div> <div>←</div> <div>→</div> <div>Least Complex</div>				
<p>division of a fraction by a fraction is not a requirement at this grade.</p> <p>a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(\frac{1}{3}) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(\frac{1}{3}) \div 4 = (\frac{1}{12})$ because $(\frac{1}{12}) \times 4 = (\frac{1}{3})$.</p> <p>b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (\frac{1}{5})$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (\frac{1}{5}) = 20$ because $20 \times (\frac{1}{5}) = 4$.</p> <p>c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, how much chocolate will each person get if 3 people share $\frac{1}{2}$ a pound of chocolate equally? How many $\frac{1}{3}$ cup servings are in 2 cups of raisins?</i></p>	3, 4, 5, 6, 8, and 10).			<ul style="list-style-type: none"> Count by unit fractions of $\frac{1}{2}$ up to a whole and onto more than one whole. Write a number sentence representing more than one whole partitioned into 2 equal shares ($\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$). Describe the shares using the words halves use the phrase half of. Describe the whole as two halves, three halves, four halves, etc. in real-world contexts. Understand a unit fraction $\frac{1}{b}$ is the smallest fractional part of a whole. Use fraction models to combine equal sized shares with like denominators. Understand you can combine equal sized shares of fractional parts of the same whole like you can combine whole numbers. Know the symbols for division (\div) and equals ($=$). Relate multiplication to division by writing an equivalent number sentence. Represent a number with a set of physical objects or a drawing. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with area (rectangles) and length (number lines) fraction models.


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex 				Least Complex
Measurement and Data				
<i>Convert like measurement units within a given measurement system.</i>				
5.MD.1 Know relative sizes of these U.S. customary measurement units: pounds, ounces, miles, yards, feet, inches, gallons, quarts, pints, cups, fluid ounces, hours, minutes, and seconds. Convert between pounds and ounces; miles and feet; yards, feet, and inches; gallons, quarts, pints, cups, and fluid ounces; hours, minutes, and seconds in solving multi-step, real-world problems.	5.MD.1a Convert, within 1-step conversion, basic units of measure for cups to pints, quarts to gallons, and inches to feet, with visual and/or physical representations to solve real-world problems.	5.MD.1b Convert basic units of measure: hours to minutes, gallons to quarts, and yards to feet using physical and/or visual representations.	5.MD.1c Match physical or visual representations of measurement tools and units (time – clock, liquid – teaspoons, cups, or gallons, ruler – feet or yards).	<p>Note for instruction, measurement in units of time (hours and minutes) do not appear in the standards after grade 5.</p> <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with measurement tools for time, length, and liquid volume.
<i>Represent and interpret data.</i>				
5.MD.2 Display and interpret data in graphs (picture graphs, bar graphs, and line plots) to solve problems using numbers and operations for this grade, e.g., including U.S. customary units in fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, or decimals.	5.MD.2a Create a line plot, bar graph, or pictograph from a given or collected data set with measurements in fractions or decimals (fourths, halves, or tenths).	5.MD.2b Create a scaled graph – picture, bar, or line graph – from given or collected data sets, and interpret the graph, including solving 1-step “how many more” and “how many less” problems.	5.MD.2c Answer questions that interpret data represented in a picture, bar, or line graph by solving one-step “how many more” and “how many less” problems.	<ul style="list-style-type: none"> Classify objects into categories. Count the number of objects in each category. Create a line graph, bar graph, or picture graph with a scale of 1 by stacking physical objects. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a line graph, bar graph, or picture graph.

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
<i>Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.</i>				
5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.	5.MD.3a Identify and measure the length, width, and height of a cube or rectangular prism to show understanding of the concept of volume (with physical or visual representations).	5.MD.3b Use a tool to fill a cube and/or rectangular prism to show understanding of volume.	5.MD.3c Match solid figure to its 2-dimensional shape counterpart with visual and/or physical representations.	<ul style="list-style-type: none"> Content works in conjunction with 5.MD.4-5. Identify and describe shapes (squares, rectangles, cubes, rectangular prisms). Identify shapes as two-dimensional (lying in a plane, “flat”) or three dimensional (“solid”). <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with linear models or physical objects (blocks).
5.MD.4 Measure volume by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	5.MD.4a Measure the volume of cubes and/or rectangular prisms by counting unit cubes.	5.MD.4b Count and write the volume of a cube by counting physical unit cubes.	5.MD.4c When given a model, student will build a cube or rectangular prism from unit cubes to show understanding of volume.	<ul style="list-style-type: none"> Content works in conjunction with 5.MD.3 and 5.MD.5. Identify and describe shapes (squares, rectangles, cubes, rectangular prisms). Identify shapes as two-dimensional (lying in a plane, “flat”) or three dimensional (“solid”). Recognize a cube and a rectangular prism. Recognize a unit cube. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with linear models or physical objects (blocks).
5.MD.5 Relate volume to the operations of multiplication and addition and solve real-world and	5.MD.5a Find the volume when given a physical	5.MD.5b Label and measure the length, width, and	5.MD.5c Label the length, width, and height of a	<ul style="list-style-type: none"> Content works in conjunction with 5.MD.3-4.


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
<p>mathematical problems involving volume.</p> <p>a. Find the volume of a right rectangular prism with whole number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole number products as volumes, e.g., to represent the Associative Property of Multiplication.</p> <p>b. Apply the formulas $V = \ell \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems.</p> <p>c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.</p>	<p>representation of a cube and/or rectangular prism in a real-world word problem.</p>	<p>height of a cube and/or rectangular prism in a real-world word problem with physical representations.</p>	<p>rectangular prism from a real-world word problem with physical representations.</p>	<ul style="list-style-type: none"> Identify and describe shapes (squares, rectangles, cubes, rectangular prisms). Identify shapes as two-dimensional (lying in a plane, “flat”) or three dimensional (“solid”). Recognize a cube and a rectangular prism. Recognize a unit cube. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with linear models or physical objects (blocks).
Geometry				
<i>Graph points on the coordinate plane to solve real-world and mathematical problems.</i>				

				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex ←————→ Least Complex				
5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond, e.g., x-axis and x-coordinate, y-axis and y-coordinate. When given a whole number ordered pair, student will plot the ordered pair using knowledge of the x-axis and y-axis.	5.G.1a When given a whole number ordered pair, student will plot the ordered pair using knowledge of the x-axis and y-axis.	5.G.1b Write the ordered pair for a plotted point in quadrant I of a coordinate plane.	5.G.1c Label the x-axis and the y-axis on quadrant I on a coordinate plane. Match a plotted point in quadrant I with its ordered pair.	<ul style="list-style-type: none"> Identify points on a horizontal number line (scale limited to whole numbers 1-10). Identify points on a vertical number line (scale limited to whole numbers 1-10). Understand a coordinate grid is formed by a vertical and horizontal number line. Recognize the horizontal number line as the x-axis. Recognize the vertical number line as the y-axis. Recognize the intersection of the x-axis and y-axis as the origin. Location of the origin is at point 0,0 on the coordinate plane. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with coordinate grid.
5.G.2 Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	5.G.2a Graph a point on a map and/or coordinate grid and use the information to solve real-world word problems.	5.G.2b Use the information on a graph to solve real-world word problems.	5.G.2c Identify a point on a map and/or coordinate grid.	<ul style="list-style-type: none"> Identify points on a horizontal number line (scale limited to whole numbers 1-10). Identify points on a vertical number line (scale limited to whole numbers 1-10). Understand a coordinate grid is formed by a vertical and horizontal number line. Recognize the horizontal number line as the x-axis. Recognize the vertical number line as the y-axis.

				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex  Least Complex				
				<ul style="list-style-type: none"> Recognize the intersection of the x-axis and y-axis as the origin. Location of the origin is at point 0,0 on the coordinate place. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with coordinate grid.
<i>Classify two-dimensional figures into categories based on their properties.</i>				
5.G.3 Identify and describe commonalities and differences between types of triangles based on angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral, and scalene triangles).	5.G.3a Identify triangles (right, scalene, and isosceles) and the size of their angles.	5.G.3b Sort triangles by angle measures or side lengths when given physical or visual representations.	5.G.3c Sort triangles by the presence or absence of right angles (physical and/or visual representations given).	<ul style="list-style-type: none"> Identify three-sided closed figures as triangles. Recognize triangles regardless of their orientations or overall size. Identify the angles of a triangle. Recognize 90° angles as right angles. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with triangles.
5.G.4 Identify and describe commonalities and differences between types of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares,	5.G.4a Identify quadrilaterals with perpendicular or parallel sides, right angles, or side lengths.	5.G.4b Sort quadrilaterals by given criteria (perpendicular sides, equal side lengths) using physical and/or	5.G.4c Given polygons with three to eight sides, identify the quadrilaterals.	<ul style="list-style-type: none"> Identify four-sided closed figures as quadrilaterals. Recognize quadrilaterals regardless of their orientations or overall size.


				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex 				Least Complex
rectangles, parallelograms, trapezoids, and rhombuses.		visual representations.		Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with quadrilaterals.

Grade 6

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex 				Least Complex
Ratio and Proportional Relationships				
Understand ratio concepts and use ratio reasoning to solve problems.				
6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote that candidate A received, candidate C received nearly three votes.”	6.RP.1a Given a visual model, represent two quantities as a ratio using whole numbers (e.g., In an image of 2 bananas and 3 oranges, what is the ratio of bananas to oranges?).	6.RP.1b Given a visual model or manipulative, identify ratios involving whole numbers (e.g., The ratio of bananas to oranges is 2 to 3. Which shows the correct ratio of bananas to oranges?).	6.RP.1c Given a manipulative, identify the units to be compared (e.g., Two bananas and three oranges are displayed. What two units are being compared?).	<ul style="list-style-type: none"> Content works in conjunction with 6.RP.2-3. Count physical objects up to 10. Sort a collection of two types of objects using a given criteria (bananas and oranges). Count the number of objects in a given group (bananas and oranges). Identify differences in two types of objects. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects.
6.RP.2 Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”	6.RP.2a Solve problems involving unit rates (e.g., If it took 2 hours to mow 6 lawns, how many lawns could be mowed in 8 hours at the same rate? At what rate were lawns being mowed?).	6.RP.2b Solve for a unit rate (e.g., It took James 2 hours to drive 100 miles. How fast did he drive per one hour?).	6.RP.2c Identify a unit rate in a word problem (e.g., James drives 65 miles per hour on the highway. How many miles does James drive in one hour?).	<ul style="list-style-type: none"> Content works in conjunction with 6.RP.1 and 6.RP.3. Recognize and understand common units of measure. <ul style="list-style-type: none"> Time: hours and minutes Length: feet and miles; meters and kilometers Liquid Capacity: cups and gallons; ounces and cups Weight: ounces and pounds Explore measurements in hours and minutes; feet and miles; meters and kilometers; cups and gallons; ounces and pounds.


				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Use different types of time measurements tools to measure time in hours and minutes. Use different types of length measurement tools to measure distance in feet and miles or meters and kilometers. Use different types of containers to measure liquid capacity in cups and gallons or ounces and cups. Use different types of scales to measure weight of objects in ounces and pounds. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with measurement tools for time, length, liquid capacity, and weight.
6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements; find missing values in the tables; and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit	6.RP.3a1 Find a missing value in a ratio table. Students may use manipulatives to find the answer. AND 6.RP.3a2 Find the percent of a number using a model.	6.RP.3b1 Build equal ratios with manipulatives and record information in a table. AND 6.RP.3b2 Find the 10%, 20%, and 30% of a number using a model.	6.RP.3c1 Build equal ratios with manipulatives. AND 6.RP.3c2 Identify or represent a percent as a rate per 100 when given a model of 100 units (e.g., Mike gave away 20 of his 100 marbles. What percent of the marbles did he give away?).	<ul style="list-style-type: none"> Content works in conjunction with 6.RP.1-2. Count physical objects up to 100 by ones and tens. Count the number of objects in a given group (bananas and oranges). Demonstrate understanding of the word "equal". Use a physical representation to skip count by 2s, 3s, 4s, 5s and 10s. Multiply one-digit numbers by 2s, 3s, 4s, 5s, and 10s. Record the number of tens and ones in a group of objects or drawings. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
<p>pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</p> <p>c. Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity; solve problems involving finding the whole, given a part and the percent.</p> <p>d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</p>				<ul style="list-style-type: none"> Understand the following as special cases: 10 can be thought of as a bundle of ten ones — called a “ten;” Understand the following as special cases: 100 can be thought of as a bundle of ten tens — called a “hundred” Understand the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). Represent a number of objects up to 100 on a hundred grid. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects.
The Number System				
<i>Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</i>				
<p>6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, create a story context for $(\frac{2}{3}) \div (\frac{3}{4})$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division</i></p>	<p>6.NS.1a Use visual models to show the relationship between the multiplication and division of fractions.</p>	<p>6.NS.1b Recognize that dividing a whole number by a fraction is separating the whole into the required fractional parts and counting how many parts are in the total</p>	<p>6.NS.1c Recognize a fraction as the division of the numerator by the denominator using unit fractions (e.g., Use a model to show that $\frac{1}{4}$ means dividing a whole into 4 equal parts).</p>	<ul style="list-style-type: none"> Recognize the symbols for division (\div) and equals ($=$). Understand a fraction represents division ($3 \div 4 = 34$). Understand that 34 can be read as 3 divided by 4. <p>Engagement Statements (demonstration of engaged in the topic)</p>

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
<i>to explain that $(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$. (In general, $(\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc}$.) How much chocolate will each person get if 3 people share $\frac{1}{2}$ pound of chocolate equally? How many $\frac{3}{4}$ cup servings are in $\frac{2}{3}$ of a cup of yogurt? How wide is a rectangular strip of land with length $\frac{3}{4}$ mi and area $\frac{1}{2}$ square mi?</i>		(e.g., Given one yard of fabric divided into pieces that are $\frac{2}{3}$ of a yard, how many pieces will there be? Use a model of solve.).		<ul style="list-style-type: none"> Interact with area (rectangles) and length (number lines) fraction models.
Compute fluently with multi-digit numbers and find common factors and multiples.				
6.NS.2 Fluently divide multi-digit numbers using a standard algorithm.	6.NS.2a Divide multi-digit whole numbers (up to 3-digit numbers) by 1- or 2-digit numbers in problems with and without remainders.	6.NS.2b Divide a 2-digit number up to 100 by a 1-digit number with and without remainders using models.	6.NS.2c Divide a 2-digit whole number up to 20 by a 1-digit whole number without remainder using models.	<ul style="list-style-type: none"> Identify 2, 3, 4, 5, 6, and 10 blocks. Identify groups of blocks 2s, 3s, 4s, 5s, 6s, and 10s. Share up to 20 objects equally between 2, 3, 4, 5, 6, and 10 people (without remainders). Build groups of blocks into rows and columns (arrays). Count the number of blocks in a given array. Identify the number of blocks in each row and each column. Recognize factors in an array. Relate a picture or objects to a number sentence. Identify a number sentence. Recognize the symbols for multiplication (\times), division (\div), and equals ($=$). Read and interpret a traditional one-step number sentence ($6 \div 3 = \square$). Know that a symbol \square can represent a missing value. Recognize related multiplication and division number sentences.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Write the number sentence to express the total as a product of two factors. Solve 1-step number sentences involving multiplication or division. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings representing multiplication and division.
6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.	6.NS.3a Add, subtract, and multiply multi-digit decimals using place value models.	6.NS.3b Add and subtract multi-digit decimals using place value models.	6.NS.3c Add decimals to the tenths place using place value models.	<ul style="list-style-type: none"> Explore place values using place value models. (Base-10 blocks, Cuisenaire rods, pennies, nickels, dimes, quarters, etc.) Know the names and values of pennies, nickels, dimes, and quarters. Know the symbols for dollars (\$), cents (¢), and decimal point (.). Match a collection of pennies and dimes to the visual model of the decimal. Record the value of a collection of pennies or dimes using dollar or cent notation. Represent a value of a collection of coins on a place value chart. Recognize the value of a decimal to tenths using a place value chart. Use language of <i>tenths</i> in real-world contexts. Add collections of coins (pennies and dimes).

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Write the number sentence for the addition of two collections of coins. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with linear models or place value models. (Base-10 blocks, Cuisenaire rods, pennies, dimes, etc.)
6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100, and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. <i>For example, express $36 + 8$ as $4(9 + 2)$.</i>	6.NS.4a1 Identify the greatest common factor of two whole numbers (up to 20). AND 6.NS.4a2 Use a factor and distributive property to rewrite the sum of two whole numbers (up to 50). Models may be used.	6.NS.4b Identify factors of whole numbers (up to 50).	6.NS.4c Identify factors of whole numbers (up to 20).	<ul style="list-style-type: none"> Identify a number sentence. Count the number of objects in an array. Recognize the symbols for addition (+), subtraction (−), multiplication (×), division (÷), and equals (=). Read and interpret a traditional one-step number sentence ($2 \times 3 = \square$). Relate a picture or objects to a number sentence. Know that a symbol \square can represent a missing value. Count to 30. Count physical objects up to 30. Identify groups of blocks 2s, 5s, and 10s. Build groups of blocks into rows and columns (arrays). Count the number of blocks in a given array. Build an array and count the number of blocks.


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←  → Least Complex				
				<ul style="list-style-type: none"> Identify the number of blocks in each row and each column. Match an array to its factors. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings representing addition, subtraction, or multiplication word problems.
<i>Apply and extend previous understandings of numbers to the system of rational numbers.</i>				
6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge. Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	6.NS.5a Represent real-world problems involving integers (e.g., temperatures, elevations, and distances from a fixed point (map reading)).	6.NS.5b Identify the opposites of real-world examples of integers (e.g., opposite of gaining 40 yards is losing 40 yards).	6.NS.5c Identify or explain positive or negative regions in real-world models (e.g., sea elevations, yardage on football field, thermometer, etc.).	<ul style="list-style-type: none"> Identify points on a horizontal number line (scale limited to whole numbers 1-10). Identify points on a vertical number line (scale limited to whole numbers 1-10). Understand a coordinate grid is formed by a vertical and horizontal number line. Recognize the horizontal number line as the x-axis. Recognize the vertical number line as the y-axis. Recognize the intersection of the x-axis and y-axis as the origin. Location of the origin is at point 0,0 on the coordinate place. <p>Engagement Statements (demonstration of engaged in the topic)</p>

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
<div> <div>Most Complex</div> <div>←</div> <div>→</div> <div>Least Complex</div> </div>				
				<ul style="list-style-type: none"> Interact with coordinate grid.
<p>6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</p> <p>a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.</p> <p>b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</p> <p>c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.</p>	<p>6.NS.6a1 Place 3 rational numbers on a number line.</p> <p>AND</p> <p>6.NS.6a2 Identify the quadrants in terms of their sign; $(+,+)$, $(+,-)$, $(-,-)$, $(-,+)$.</p>	<p>6.NS.6b1 Find an integer and its opposite on a number line.</p> <p>AND</p> <p>6.NS.6b2 Explain the directionality of the x- and y- axis (horizontal vs. vertical).</p>	<p>6.NS.6c1 Locate a given positive or negative number on a number line.</p> <p>AND</p> <p>6.NS.6c2 Identify the quadrants of a coordinate plane as Quadrant I, Quadrant II, Quadrant III, Quadrant IV.</p>	<ul style="list-style-type: none"> Know what a number line is. Know the order of the numbers from 0 to 10. Identify a whole number on a number line marked with whole numbers up to 10. Identify 0 on a number line. Identify a missing whole number value on a number line marked with whole number up to 10. Compare distances of objects using a vertical or horizontal number line. Understand that 2 is the distance from 0 to 2 and 3 is the distance from 0 to 3 using standard units for all lengths from 1 to 10. Identify points on a horizontal number line (scale limited to whole numbers 1-10). Identify points on a vertical number line (scale limited to whole numbers 1-10). Understand a coordinate grid is formed by a vertical and horizontal number line. Recognize the horizontal number line as the x-axis. Recognize the vertical number line as the y-axis. Recognize the intersection of the x-axis and y-axis as the origin. Location of the origin is at point 0,0 on the coordinate place.

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
<div> <div>Most Complex</div> <div>←</div> <div>→</div> <div>Least Complex</div> </div>				
				Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with coordinate grid.
6.NS.7 Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. <i>For example, write $-3^{\circ}\text{C} > -7^{\circ}\text{C}$ to express the fact that -3°C is warmer than -7°C.</i> c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. <i>For example, for an account balance of -30 dollars, write $-30 = 30$ to describe the size of the debt in dollars.</i> d. Distinguish comparisons of absolute value from statements about order. For example,	6.NS.7a On a number line, order rational numbers from smallest to largest (limit to 3 rational numbers).	6.NS.7b On a number line, order integers from smallest to largest (limit to 5 whole numbers).	6.NS.7c On a number line, order whole numbers from smallest to largest (limit to 3 whole numbers).	<ul style="list-style-type: none"> Identify points on a horizontal number line (scale limited to whole numbers 1-10). Identify points on a vertical number line (scale limited to whole numbers 1-10). Understand a coordinate grid is formed by a vertical and horizontal number line. Recognize the horizontal number line as the x-axis. Recognize the vertical number line as the y-axis. Recognize the intersection of the x-axis and y-axis as the origin. Location of the origin is at point 0,0 on the coordinate place. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with coordinate grid.

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.				
6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	6.NS.8a Plot whole number ordered pairs in a real-world example (e.g., mapping locations).	6.NS.8b Plot whole number ordered pairs.	6.NS.8c Identify the x- and y-axis, and plot an ordered pair in quadrant 1 of a coordinate plane.	<ul style="list-style-type: none"> Content works in conjunction with 6.EE.9. Identify points on a horizontal number line (scale limited to whole numbers 1-10). Identify points on a vertical number line (scale limited to whole numbers 1-10). Understand a coordinate grid is formed by a vertical and horizontal number line. Recognize the horizontal number line as the x-axis. Recognize the vertical number line as the y-axis. Recognize the intersection of the x-axis and y-axis as the origin. Location of the origin is at point 0,0 on the coordinate place. Identify points on a horizontal number line (scale limited to whole numbers 1-10). Identify points on a vertical number line (scale limited to whole numbers 1-10). Understand a coordinate grid is formed by a vertical and horizontal number line. Recognize the horizontal number line as the x-axis. Recognize the vertical number line as the y-axis. Recognize the intersection of the x-axis and y-axis as the origin. Location of the origin is at point 0,0 on the coordinate place.

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with coordinate grid.
Expressions and Equations				
<i>Apply and extend previous understandings of arithmetic to algebraic expressions.</i>				
6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.	6.EE.1a Write and evaluate numerical expressions involving exponents of squares and cubes only (e.g., $5^2 + 4^3$ evaluate). No variables should be used.	6.EE.1b Write and/or evaluate expressions with integers (e.g., a model of 10 apples and giving 2 away). No variables or exponents should be used.	6.EE.1c Identify a model that is equivalent to a numerical expression.	<ul style="list-style-type: none"> Content works in conjunction with 6.EE.2-9. Identify a number sentence. Identify a physical object or drawing. Know the operations for each of the symbols (+, -, ×, ÷, =) Read and interpret a traditional number sentence ($2 \times 2 = 4$) Relate a picture or objects to a number sentence. Group or partition sets of numbers into equal groups to determine the missing number. Record a number sentence to express the total as the sum of equal addends. Identify a numerical expression without unknowns or exponents. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings that represents an expression.

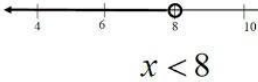
				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
<i>Apply and extend previous understandings of arithmetic to algebraic expressions.</i>				
<p>6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.</p> <p>a. Write expressions that record operations with numbers and with letters standing for numbers. <i>For example, express the calculation "Subtract y from 5" as $5 - y$.</i></p> <p>b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. <i>For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.</i></p> <p>c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. <i>For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.</i></p>	<p>6.EE.2a Given a context, student will write an algebraic expression for a context with 2 or 3 terms involving variables.</p>	<p>6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient) in simple expressions.</p>	<p>6.EE.2c Evaluate an algebraic expression with 2 to 3 terms. (The value of the variables to be substituted into the expression should be limited to whole numbers.)</p>	<ul style="list-style-type: none"> Content works in conjunction with 6.EE.1 and 6.EE.3-9. Identify a number sentence. Know the operations for each of the symbols (+, -, ×, ÷, =) Read and interpret a traditional one-step number sentence ($2 \times 3 = _$). Relate a picture or objects to a number sentence. Know that a letter or a symbol can represent a missing value. Recognize a numerical expression with and without variables. Identify a numerical expression without exponents. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings that represents an expression.

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
6.EE.3 Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.	6.EE.3a Identify equivalent algebraic expressions using distributive property (e.g., $2(x+3)$ is equivalent to $2x+6$).	6.EE.3b Identify equivalent algebraic expressions using commutative property (e.g., $x+3$ is equivalent to $3+x$).	6.EE.3c Identify equivalent numerical expressions using the commutative property (e.g., $2+3$ is equivalent to $3+2$).	<ul style="list-style-type: none"> Content works in conjunction with 6.EE.1-2 and 6.EE.4-9. Identify a number sentence. Know the operations for each of the symbols (+, -, ×, ÷, =) Read and interpret a traditional number sentence ($2 \times 3 = 6$). Relate a picture or objects to a number sentence. Write the number sentence to express the total as a product of two factors. Know that a letter or a symbol can represent a missing value. Recognize a numerical expression with and without variables. Identify a numerical expression without exponents. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings that represents an expression.
6.EE.4 Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them. <i>For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the</i>	6.EE.4a Identify equivalent expressions (e.g., $2x + x$ is equivalent to $3x$) with whole number coefficients.	6.EE.4b Identify equivalent expressions (limit to variables with no coefficient) (e.g., $x+x$ is equivalent to $2x$).	6.EE.4c Identify equivalent expressions (limit to whole number expressions) (e.g., $2+3$ is equivalent to 5).	<ul style="list-style-type: none"> Content works in conjunction with 6.EE.1-3 and 6.EE.5-9. Identify a number sentence. Know the operations for each of the symbols (+, -, ×, ÷, =) Read and interpret a traditional number sentence ($2 \times 3 = 6$). Relate a picture or objects to a number sentence.


				Learning Progression Building the Base & Engagement
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Most Complex ←————→ Least Complex				
<i>same number regardless of which number y stands for.</i>				<ul style="list-style-type: none"> Write the number sentence to express the total as a product of two factors. Know that a letter or a symbol can represent a missing value. Recognize a numerical expression with and without variables. Identify a numerical expression without exponents. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings that represents an expression.
<i>Reason about and solve one-variable equations and inequalities.</i>				
6.EE.5 Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	6.EE.5a Given an inequality statement in the form of $x > c$ or $x < c$ determines if a given value makes the inequality true. (For $3 > b$, if b equals 5, is the inequality true?)	6.EE.5b Given a one-step equation and set of values for the variable, determine which value makes the equation true. (For $3x = 9$, does $x = 2$, 3, 4, or 6 make the equation true?)	6.EE.5c Given a one-step equation and a value for the variable, determine if the value makes the equation true. (For $3x = 9$, does $x = 4$ make the equation true?)	<ul style="list-style-type: none"> Content works in conjunction with 6.EE.1-4 and 6.EE.6-9. Identify a number sentence. Know the operations for each of the symbols (+, -, ×, ÷, =) Read and interpret a traditional one-step number sentence ($2 \times 3 = _$). Relate a picture or objects to a number sentence. Know that a letter or a symbol can represent a missing value. Recognize a numerical expression with and without variables.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Identify a numerical expression without exponents. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings that represents an expression.
6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	6.EE.6a Represent the missing information with a variable in a real-world problem.	6.EE.6b Identify the missing information in real-world problem.	6.EE.6c Identify a variable in an equation.	<ul style="list-style-type: none"> Content works in conjunction with 6.EE.1-5 and 6.EE.7-9. Identify a number sentence. Know the operations for each of the symbols (+, -, ×, ÷, =) Read and interpret a traditional one-step number sentence ($2 \times 3 = _$). Relate a picture or objects to a number sentence. Know that a letter or a symbol can represent a missing value. Recognize a numerical expression with and without variables. Identify a numerical expression without exponents. <p>Engagement Statements (demonstration of engaged in the topic)</p>

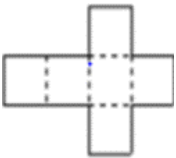
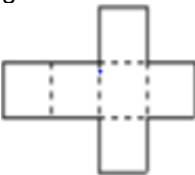
				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings that represents an expression.
6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q , and x are all non-negative rational numbers.	6.EE.7a Using real-world contexts, write and solve one-step equations using whole numbers. Models may be used (e.g., Jim has several balloons. He gives 2 balloons to his brother and now has 5 balloons left. How many balloons did Jim have to begin with?).	6.EE.7b Solve a one-step equation using one of the 4 operations with whole numbers using models.	6.EE.7c Solve a one-step equation using addition and subtraction with whole numbers using models.	<ul style="list-style-type: none"> Content works in conjunction with 6.EE.1-6 and 6.EE.8-9. Know what a number line is. Know the order of the numbers from 0 to 50. Identify a whole number on a number line marked with whole numbers up to 50. Identify 0 on a number line. Identify a missing whole number value on a number line marked with whole number up to 50. Compare distances of objects using a vertical or horizontal number line. Understand that 2 is the distance from 0 to 2 and 3 is the distance from 0 to 3 using standard units for all lengths from 1 to 50. Represent a number with a set of physical objects or a drawing. Understand addition is the combining of two (or more) sets of objects. Understand subtraction is taking away of one amount of objects from another. Understand that addition and subtraction are opposites. Recognize the symbols for addition (+), subtraction (−), and equals (=). Relate counting to addition and subtraction, e.g., by counting on 2 to add 2. Add and subtract within 10 using strategies. Strategies may include:

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Counting on Making ten ($8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$) Decomposing a number leading to a ten ($13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$) Using the relationship between addition and subtraction; knowing that <p>$8 + 4 = 12$, one knows $12 - 8 = 4$ and</p> <ul style="list-style-type: none"> Creating equivalent but easier or known sums (adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$). <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects (blocks) or drawings (may include 100s chart) representing whole numbers within 50.
6.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	6.EE.8a Identify an inequality that is represented on a number line using a variable (e.g.,  $x < 8$	6.EE.8b Identify an inequality that represents a real-world or mathematical problem (e.g., Jane is 23 years old. Sally is 19 years old. Which inequality correctly	6.EE.8c Identify an inequality that compares two whole numbers using the $>$, $<$, and $=$.	<ul style="list-style-type: none"> Content works in conjunction with 6.EE.1-7 and 6.EE.9. Recognize the numerals from 1 to 100. Represent numbers from 1 to 100 using physical objects. Know the word names for the numbers 1-100. Write numerals from 0 to 100. Know the order of numbers from 1 to 100.

				Learning Progression <i>Building the Base & Engagement</i>								
Learning Standard	Complexity a	Complexity b	Complexity c									
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		compares Jane and Sally's ages? 23>19).		<ul style="list-style-type: none">Relate the understanding of order to the concept of <i>greater than</i> or <i>less than</i>.<ul style="list-style-type: none">5 apples is less than 7 applesExplore place value tools.<ul style="list-style-type: none">Base-10 blocksPlace value chart100's chart <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none">Interact with physical objects (blocks) or drawings.								
Represent and analyze quantitative relationships between dependent and independent variables.												
6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent</i>	6.EE.9a Complete a table given a one-step equation and graph the coordinates.	6.EE.9b Find the outputs (dependent variable) given an equation and the inputs in table form. $y = 3 + x$ <table><tr><td>x</td><td>y</td></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>5</td></tr><tr><td>3</td><td>6</td></tr></table>	x	y	1	4	2	5	3	6	6.EE.9c Identify the input and output in a table.	<ul style="list-style-type: none">Content works in conjunction with 6.NS.8 and 6.EE.1-8.Explore a chart or a table.Know the meaning of the words <i>input</i> and <i>output</i>.Relate <i>input</i> and <i>output</i> to a function machine.Understand that a table is made up of columns and rows. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none">Interact with a chart or table.
x	y											
1	4											
2	5											
3	6											

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
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<i>the relationship between distance and time.</i>				
Geometry				
<i>Solve real-world and mathematical problems involving area, surface area, and volume.</i>				
6.G.1 Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems.	6.G.1a Demonstrate that the area of a right triangle is $\frac{1}{2} \times \text{length} \times \text{height}$ (e.g., Two same right triangles combined make a rectangle, and the area of a triangle is half the area of the rectangle it can be composed into). Demonstrate these techniques in real-world and mathematics problems.	6.G.1b Demonstrate that the area of all rectangles is $\text{length} \times \text{width}$ (e.g., Multiply side lengths to find the area of rectangles with whole-number side lengths). Demonstrate these techniques in real-world and mathematics problems.	6.G.1c Find the area of rectangles with whole-number side lengths by counting unit squares. Demonstrate these techniques in real-world and mathematics problems.	<ul style="list-style-type: none"> Identify surfaces where an area can be measured. Lay unit squares on a flat surface. Count unit squares on a flat surface. Arrange unit squares into rows and columns. Identify surfaces where a perimeter can be measured. Lay inch squares around a flat surface. Count inch squares all the way around a flat surface. Measure length and width with units and record the measurement. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with flat two-dimensional surfaces.
6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = \ell \cdot w \cdot h$ and $V = B \cdot h$	6.G.2a Recognize that the volume of a right rectangular prism can be found by multiplying the height by the area of the base (using whole numbers) (e.g., show that $V =$	6.G.2b Demonstrate that unit cubes can be used to build figures that have volume, and determine the volume of a figure. Limit to whole	6.G.2c Find the volume of a right rectangular prism (e.g., count the number of unit cubes it takes to fill a rectangular prism) (up to 25 cubes). Limit to	<ul style="list-style-type: none"> Identify and describe shapes (squares, rectangles, cubes, rectangular prisms). Identify shapes as two-dimensional (lying in a plane, "flat") or three dimensional ("solid"). Recognize a cube and a rectangular prism. Recognize a unit cube. Understand a solid figure which can be packed without gaps or overlaps using n unit

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Most Complex ←————→ Least Complex				
to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	$\ell \cdot w \cdot h$ and $V = B \cdot h$. Limited to whole number edge lengths.	number edge lengths.	whole number edge lengths.	cubes is said to have a volume of n cubic units. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with linear models or physical objects (blocks).
6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	6.G.3a Find the length(s) of the side(s) of a polygon drawn in quadrant 1 of the coordinate plane.	6.G.3b Plot points of a polygon in quadrant 1 of the coordinate plane and identify the name of the shape.	6.G.3c Given a graph of plotted vertices, connect the vertices forming a polygon and identify the name of the shape that is created.	<ul style="list-style-type: none"> Identify points on a horizontal number line (scale limited to whole numbers 1-10). Identify points on a vertical number line (scale limited to whole numbers 1-10). Understand a coordinate grid is formed by a vertical and horizontal number line. Recognize the horizontal number line as the x-axis. Recognize the vertical number line as the y-axis. Recognize the intersection of the x-axis and y-axis as the origin. Location of the origin is at point 0,0 on the coordinate place. Identify points on a horizontal number line (scale limited to whole numbers 1-10). Identify points on a vertical number line (scale limited to whole numbers 1-10). Understand a coordinate grid is formed by a vertical and horizontal number line. Recognize the horizontal number line as the x-axis. Recognize the vertical number line as the y-axis.

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Recognize the intersection of the x-axis and y-axis as the origin. Location of the origin is at point 0,0 on the coordinate place. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with coordinate grid.
6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	6.G.4a Given a net and a three-dimensional figure, find the surface area of prisms, pyramids, and cubes. 	6.G.4b Identify a net given a three-dimensional figure or identify three-dimensional figure given a net. 	6.G.4c Identify cubes, rectangular prisms (e.g., cubes, rubber eraser, pyramids).	<ul style="list-style-type: none"> Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, rectangular prisms, and spheres). Know the names of three-dimensional shapes (cubes, cones, cylinders, rectangular prisms, and spheres). Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. Correctly name shapes regardless of their orientations or overall size. Identify shapes as two-dimensional (lying in a plane, “flat”) or three dimensional (“solid”). Describe, compare, create, and compose shapes. Describe and compare two- or three-dimensional shapes, in different sizes and orientations, using informal language to describe their commonalities, differences, parts, and other attributes.

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Model shapes in the world by building shapes from components (such as sticks and clay balls) and drawing shapes. Combine simple shapes to form larger shapes. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with three-dimensional objects in their environment.
Statistics and Probability				
<i>Develop understanding of statistical problem solving.</i>				
6.SP.1 Develop statistical reasoning by using the GAISE model: a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. <i>For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because of the variability in students' ages. (GAISE Model, step 1)</i> b. Collect Data: Design and use a plan to collect appropriate data to	6.SP.1a Recognize a statistical question.	6.SP.1b Identify when we might pose a statistical question.	6.SP.1c Ask questions about a statistical situation.	<ul style="list-style-type: none"> Identify a question. Know the meaning of the question mark (?). Organize, represent, and interpret data with up to four categories. Create picture graphs and bar graphs using whole number unit scales larger than one. Ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. (limit to whole numbers). Understand the difference between a question and a statistical question. <p>Engagement Statements (demonstration of engaged in the topic)</p>

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
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answer a statistical question. (GAISE Model, step 2) c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) d. Interpret Results: Draw logical conclusions from the data based on the original question. (GAISE Model, step 4)				<ul style="list-style-type: none"> Interact with data.
6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	6.SP.2a Given a data display, answer a statistical question about spread.	6.SP.2b Given a data display, answer a statistical question about center.	6.SP.2c Given a data display, answer a statistical question about shape.	<ul style="list-style-type: none"> Identify information in a chart, table, or graph. Given a bar or picture graph build a graph based on student sorted data for example, votes for 4 different candidates or weather types, or occurrences of event or behavior). Interpret data from a given line, picture, or bar graph to solve a multi-step problem (limit to whole numbers). Interpret data represented in a graph by solving one-step “how many more” and “how many less” problems (limit to whole numbers). <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with data.

				Learning Progression Building the Base & Engagement
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	6.SP.3a1 Find the mean using models according to the idea of “fair share.” Limit to 7 data points. AND 6.SP.3a2 Find the median of a data point. Limit to 7 data points.	6.SP.3b Find the median of a data set with an odd number of data points. Limit to 7 data points.	6.SP.3c Identify the mode for a set of data.	<ul style="list-style-type: none"> Identify numbers in a set of data. Identify the largest value in a data set. Identify information in a chart, table, or graph. Given a bar graph or picture graph, identify the largest/tallest data point. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with data.
Summarize and describe distributions.				
6.SP.4 Display numerical data in plots on a number line, including dot plots (line plots), histograms, and box plots. (GAISE Model, step 3)	6.SP.4a Given pre-made axes, construct and interpret a histogram from a given or collected data set.	6.SP.4b Given pre-made axes, construct and analyze a line plot from a given or collected data set.	6.SP.4c Given pre-made axes, construct and analyze a bar graph from a given or collected data set.	<ul style="list-style-type: none"> Identify information in a chart, table, or graph. Given a bar or picture graph build a graph based on student sorted data. Sort the categories by count. Interpret data from a given line, picture, or bar graph to solve a multi-step problem (limit to whole numbers). Interpret data represented in a graph by solving one-step “how many more” and “how many less” problems (limit to whole numbers). <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with data.

				Learning Progression Building the Base & Engagement
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Most Complex ←————→ Least Complex				
6.SP.5 Summarize numerical data sets in relation to their context. a. Report the number of observations. b. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range), as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution. d. Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered.	6.SP.5a Interpret information from a given or collected data set (e.g., Given a tally chart showing the number of pockets on students' clothes in a class. Find the average number of pockets and the range of the data).	6.SP.5b Interpret information from a given or collected data set (e.g., Given a tally chart showing the number of pets. Find the average number of pets).	6.SP.5c Interpret information from a given or collected data set (e.g., given a tally chart showing the favorite colors of the students in Joe's math class, determine which color was the most/least favorite).	<ul style="list-style-type: none"> Identify information in a chart, table, or graph. Given a bar or picture graph build a graph based on student sorted data. Sort the categories by count. Interpret data from a given line, picture, or bar graph to solve a multi-step problem (limit to whole numbers). Organize, represent, and interpret data with up to four categories. Create picture graphs and bar graphs using whole number unit scales larger than one. Ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. (limit to whole numbers). Solve simple put together, take-apart, and compare problems with whole numbers in a graph. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with data.

Grade 7

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
Ratio and Proportional Relationships				
<i>Analyze proportional relationships and use them to solve real-world and mathematical problems.</i>				
7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. <i>For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $(\frac{1}{2})/(\frac{1}{4})$ miles per hour; equivalently 2 miles per hour.</i>	7.RP.1a Given a model or pictures of a ratio, build the unit rate (e.g., given 12 pieces of candy for \$3, find the unit rate).	7.RP.1b Given a model of a unit rate, build equivalent ratios (e.g., every 4 pieces of candy cost \$1. Using candy and play \$1 bills, build equivalent ratios.).	7.RP.1c Given models of equivalent ratios, identify the unit rate. Using candy and play \$1 bills, the student is shown 6 candies for \$2, 9 candies for \$3, and 3 candies for \$1. Identify the unit rate.	<ul style="list-style-type: none"> Count physical objects up to 10. Sort a collection of two types of objects using a given criteria (bananas and oranges). Count the number of objects in a given group (bananas and oranges). Identify differences in two types of objects. Identify the value of dollars. Know the symbol for dollars (\$). Know that unit rate means per (for) every 1. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects. Interact with dollar bills.
7.RP.2 Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	7.RP.2a1 Given a graph of a proportion, find an ordered pair on the line. 7.RP.2a2 Using models, determine whether two quantities represent a proportion.	7.RP.2b1 Given three ordered pairs that represents a proportional relationship, plot them on a coordinate grid and connect the line. 7.RP.2b2 Find a missing value in a	7.RP.2c1 Identify if a graph represents a proportional relationship. 7.RP.2c2 Build a proportion with objects such as blocks and record	<ul style="list-style-type: none"> Identify points on a horizontal number line (scale limited to whole numbers 1-10). Identify points on a vertical number line (scale limited to whole numbers 1-10). Understand a coordinate grid is formed by a vertical and horizontal number line. Recognize the horizontal number line as the x-axis. Recognize the vertical number line as the y-axis.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.		ratio table. Students may use manipulatives to find the answer.	the information in a table.	<ul style="list-style-type: none"> Recognize the intersection of the x-axis and y-axis as the origin. Identify a line. Identify the point $(0,0)$ on a graph. Represent a numeral with physical objects. Recognize and compare differences between two sets of objects. Represent two sets of different objects. Count physical objects up to 100 by ones and tens. Count the number of objects in a given group (bananas and oranges). Demonstrate understanding of the word "equal". Understand that a table is made up of columns and rows. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects. Interact with two different sets of physical objects. Interact with a graph or table.
7.RP.3 Use proportional relationships to solve multi-step ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	7.RP.3a Find the percent of a number in real-world problem involving tax or gratuity.	7.RP.3b Find the percent of a number.	7.RP.3c Find 10%, 20%, and 30% of a number given a model of 100 units.	<ul style="list-style-type: none"> Count physical objects up to 100 by ones and tens. Represent a number of objects up to 100 on a hundred grid. Use a physical representation to skip count by 10s. Recognize the percent symbol. Identify the percent symbol on a calculator.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←				→ Least Complex
				<ul style="list-style-type: none"> Demonstrate finding a percent on a calculator. Know that percent is out of 100. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a calculator. Interact with a model.
The Number System				
<i>Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</i>				
<p>7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</p> <p>a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.</p> <p>b. Understand $p + q$ as the number located a distance q from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a</p>	<p>7.NS.1a Add and subtract integers.</p>	<p>7.NS.1b1 Recognize that the absolute value of an integer is how far it is from 0 on the number line (e.g., plot a number and its opposite on a number line and recognize that they are equidistant from zero).</p> <p>7.NS.1b2 Add and subtract whole numbers using models.</p>	<p>7.NS.1c1 Recognize that addition means move to the right and subtraction means move to the left on a number line.</p> <p>7.NS.1c2 Add whole numbers using models.</p>	<ul style="list-style-type: none"> Know what a number line is. Know the order of the numbers from 0 to 10. Identify a whole number on a number line marked with whole numbers up to 10. Identify 0 on a number line. Identify the symbols (+) and (-). <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a model.


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
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sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.				
7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of	7.NS.2a Multiply and divide integers using models.	7.NS.2b Multiply and divide whole numbers using models.	7.NS.2c Multiply whole numbers using models.	<ul style="list-style-type: none"> Create and count multiple groups of using objects. Repeatedly add groups using physical objects. Understand the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). Know the symbols for multiplication (\times) and equals ($=$). Relate a picture or objects to a number sentence. Recognize and identify the (\times) and ($=$) keys on a calculator. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with a calculator.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
<p>integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.</p> <p>c. Apply properties of operations as strategies to multiply and divide rational numbers.</p> <p>d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.</p>				<ul style="list-style-type: none"> Interact with a model. Interact with physical objects or other representations of multiplication.
<p>7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.</p>	<p>7.NS.3a Multiply fractions when solving real-world and mathematical problems using models.</p>	<p>7.NS.3b Add and subtract fractions with same/unlike denominator when solving real-world and mathematical problems using models.</p>	<p>7.NS.3c Add fractions with same denominator when solving real-world and mathematical problems using models.</p>	<ul style="list-style-type: none"> Identify a unit fraction ($1/4$ or $1/2$) as part of a whole when shown as a physical and/or visual representation. Identify the symbols (+) and (=). Recognize that in a fraction the top number is the numerator and the bottom number is the denominator. Identify the same sized whole partitioned into 2, 3, 4, 5, 6, 8, and 10 equal shares. Recognize the value of a whole fractional shaded part. Identify $1/2$, $1/3$, $1/4$, etc. when given a fraction model. Write a number sentence representing a whole partitioned into 2, 3, 4, 5, 6, 8, or 10 equal shares ($1/4 = 1/4 + 1/4 + 1/4 + 1/4$). Count by unit fractions up to a whole. ($1/2$, $1/3$, $1/4$, etc.)

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Use fraction models to combine equal sized shares with like denominators. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with fraction models.
Expressions and Equations				
<i>Use properties of operations to generate equivalent expressions.</i>				
7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	7.EE.1a Apply the order of operations to problems using whole numbers. Limit the number of terms to 4.	7.EE.1b Apply the first step of the order of operations to create an equivalent expression. Limit the number of terms to 3.	7.EE.1c Identify the first step to complete the order of operations (e.g., $2(3 + 5) - 10 + 2$, what is the first step? Add $3 + 5$.) Limit the number of terms to 3.	<ul style="list-style-type: none"> Identify a number sentence. Recognize the symbols for addition (+), subtraction (−), multiplication (×), division (÷), and equals (=). Read and interpret a traditional one-step number sentence ($2 \times 3 =$). Relate a picture or objects to a number sentence. Know that a symbol can represent a missing value. Know the symbols () Know parentheses are evaluated first within an expression. Demonstrate understanding when there can be more than one step to solve a problem. Engagement Statements (demonstration of engaged in the topic)

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Interact with linear models or physical objects or drawings representing addition, subtraction, multiplication, or division.
7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. <i>For example, a discount of 15% (represented by $p - 0.15p$) is equivalent to $(1 - 0.15)p$, which is equivalent to $0.85p$ or finding 85% of the original price.</i>	7.EE.2a Create an equivalent expression by giving one missing term (limit to addition, subtraction, and multiplication, using whole numbers) (e.g., $6 \times 4 = 8 \times ?$).	7.EE.2b Create an equivalent expression by giving one missing term (limit to addition and subtraction using whole numbers) (e.g., $7 + 1 = 6 + ?$).	7.EE.2c Identify equivalent expressions (limit to addition using whole numbers) (e.g., $5 + 2 = 6 + 1$).	<ul style="list-style-type: none"> Demonstrate understanding of the word "equal". Recognize the symbols for addition (+). Relate a picture or objects to a number sentence. Know that more than one number can be to the right of an (=) symbol. Know that there is more than one way to represent a number. Decompose a number in more than one combination. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with physical objects or drawings representing addition.
<i>Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</i>				
7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any	7.EE.3a Solve one-step real-life and mathematical problems (limit to fractions) (e.g., the recipe for 12 cupcakes asks for $\frac{2}{3}$ cup of sugar. How	7.EE.3b Solve one-step real-life and mathematical problems (limit to decimals) (e.g., Sue spends \$2.35 on a notebook and \$1.60 on a ruler. How much	7.EE.3c Solve one-step real-life and mathematical problems (limit to whole numbers) (e.g., Jim spends \$3 on a pen and	<ul style="list-style-type: none"> Identify a number sentence. Recognize the symbols for addition (+), subtraction (-), multiplication (x), division (÷), and equals (=) and identify the keys on a calculator. Read and interpret a traditional one-step number sentence in a context ($2 \times 3 =$).

				Learning Progression <i>Building the Base & Engagement</i>
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form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. <i>For example, if a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</i>	many cups of sugar is needed if the recipe is doubled?).	does Sue spend in all?).	\$2 on a pencil. How much does Jim spend in all?).	<ul style="list-style-type: none"> Relate a picture or objects to a number sentence. Know that a symbol can represent a missing value. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with linear models or physical objects or drawings representing addition, subtraction, multiplication, or division. Interact with a calculator.
7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. <i>For example, the perimeter of a rectangle is 54 cm.</i>	7.EE.4a Use variables to show and solve a real-world or mathematical problem (limit to two-step problems involving whole numbers and one variable) (e.g., Mary pays a \$5 flat rate plus a \$2 hourly rate for each hour, x , for parking. Mary has \$15. Which equation should Mary use to calculate the total number of hours she can park? $2x + 5 =$	7.EE.4b Use variables to solve a real-world or mathematical problem (limit to two-step problems and one variable) (e.g., Mary pays a \$5 flat rate plus a \$2 hourly rate for each hour, x , for parking. Mary has \$15 is represented by $2x + 5 = 15$; solve for x).	7.EE.4c Use variables to show a real-world or mathematical problem (limit to one-step problems involving whole numbers and one variable) (e.g., Mary has \$15. She buys a bag of apples for \$4. Which equation shows how much money, x , Mary	<ul style="list-style-type: none"> Know that a letter or a symbol can represent a missing value. Recognize a numerical expression with and without variables. Identify a number sentence. Recognize the symbols for addition (+), subtraction (−), multiplication (×), division (÷), and equals (=) and identify the keys on a calculator. Read and interpret a traditional one-step number sentence in a context ($2 \times 3 =$). Relate a picture or objects to a number sentence. <p>Engagement Statements (demonstration of engaged in the topic)</p>


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
<p><i>Its length is 6 cm. What is its width?</i></p> <p>b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. <i>For example, as a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.</i></p>	<p>15, $5x + 2 = 15$, $2 + 5 + x = 15$, or $15 + 5 + 2 = x$).</p>		<p>has left? Key: $x = 15 - 4$).</p>	<ul style="list-style-type: none"> Interact with linear models or physical objects or drawings representing addition, subtraction, multiplication, or division. Interact with a calculator.
Geometry				
<i>Draw, construct, and describe geometrical figures and describe the relationships between them.</i>				
<p>7.G.1 Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals.</p> <p>a. Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale.</p> <p>b. Represent proportional relationships within and between similar figures.</p>	<p>7.G.1a Solve problems involving scaled drawings of figures (e.g., if a triangle is drawn on a grid, what will be the length of one of the sides if the triangle is increased by a factor of 2?).</p>	<p>7.G.1b Identify similar geometric figures on a grid (e.g., which shape is twice the size of another shape?).</p>	<p>7.G.1c Identify same size/same shape polygons drawn on a grid (e.g., square, rectangles, quadrilaterals, isosceles triangles, right triangles, scalene triangles, and obtuse triangles).</p>	<p>** Note** Congruent should be referred to as same size same shape or equal in measure.</p> <p>A simple grid should be used in place of a coordinate plane.</p> <ul style="list-style-type: none"> Understand grids are formed by vertical and horizontal lines. Recognize 2D shapes regardless of orientation. Recognize there are different types of triangles and quadrilaterals. Identify the same sized angles. Count squares on a grid.


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Understand that the squares are used to measure the size of the shape. Recognize by measuring or counting side lengths that are the same. Count the number of sides in a shape. Differentiate between 3 sided and 4 sided shapes. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with grid. Interact with 2D shapes.
7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. a. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. b. Focus on constructing quadrilaterals with given conditions, noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions.	7.G.2a Analyze special quadrilaterals and triangles a. by the measure of the angles (acute, obtuse, and right). by the measures of their side lengths (isosceles, equilateral, and scalene triangles; parallelogram, rhombus, and trapezoid).	7.G.2b Identify and recognize special quadrilaterals or triangles by using parallel and perpendicular sides.	7.G.2c Identify the type of angles in a triangle and the angles in a special quadrilateral.	<ul style="list-style-type: none"> Understand that 90 degrees (right angle) is marked by a square. Recognize that a right angle forms a square corner, an acute angle is smaller than a square corner, and an obtuse angle is larger than a square corner. Recognize that a right angle is 90 degrees, acute angle is less than 90 degrees, and an obtuse angle is more than 90 degrees. Recognize that a triangle has 3 angles and a quadrilateral has 4 angles. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with shapes in their environment.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	7.G.3a Using models, identify two-dimensional shapes that result from slicing a three-dimensional figure (limit to prisms and horizontal and vertical cuts).	7.G.3b Identify the shape of two-dimensional faces of three dimensional figures (limit to rectangular prisms and cubes).	7.G.3c Identify, by naming, two- and three-dimensional figures (manipulatives can be used).	<ul style="list-style-type: none"> Recognize the differences between 2D and 3D shapes. Identify shapes as two-dimensional (lying in a plane, flat) or three-dimensional (solid). Name shapes regardless of their orientations or overall size. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with two- and three-dimensional objects in their environment.
<i>Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.</i>				
7.G.4 Work with circles. a. Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. b. Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems.	7.G.4a1 Apply the formula for finding the area of a circle when given the radius. 7.G.4a2 Measure diameters and circumference of various circles to show the relationship is close to 3.14.	7.G.4b Identify the attributes of a circle (radius, diameter, circumference, and center).	7.G.4c1 Identify circles in the environment.	<ul style="list-style-type: none"> Understand that circles are round. Understand that circles have no angles. Understand that circles have no sides. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with circles in their environment.
7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	7.G.5a Identify unknown angles and solve problems when using facts about adjacent and vertical angles using visual models.	7.G.5b Classify angles as supplementary, complementary, vertical, or adjacent using visual models.	7.G.5c Sort angles by type using visual models (right, acute, obtuse, straight).	<ul style="list-style-type: none"> Understand that 90 degrees (right angle) is marked by a square. Recognize that a right angle forms a square corner, an acute angle is smaller than a square corner, and an obtuse angle is larger than a square corner.

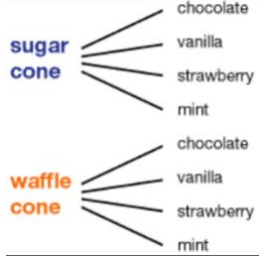
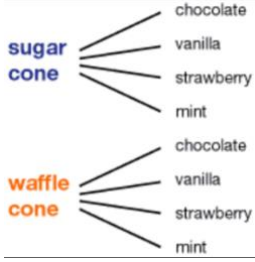
				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Recognize that a right angle is 90 degrees, acute angle is less than 90 degrees, and an obtuse angle is more than 90 degrees. Recognize that a straight angle makes a straight line and is 180 degrees. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with angles in the environment.
7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	7.G.6a1 Solve real-world problems involving surface area of a prism, cube, and pyramid. Use whole number edge lengths. 7.G.6a2 Solve real-world problems involving finding the volume of a right prism or cube. Use whole number edge lengths.	7.G.6b Solve real-world problems involving the area of figures involving rectangles and right triangles (manipulatives can be used).	7.G.6c Solve real-world problems involving perimeter (manipulatives can be used).	<ul style="list-style-type: none"> Identify surfaces where a perimeter can be measured. Lay inch squares up to 20 around a flat surface. Count inch squares up to 20 all the way around a flat surface. Measure length and width with units and record the measurement. Perimeter is found by adding the measures of all the sides. Recognize the symbols for addition (+) and equals (=) and identify the keys on a calculator. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with flat two-dimensional surfaces. Interact with a calculator.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
Statistics and Probability				
<i>Use sampling to draw conclusions about a population.</i>				
7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population. a. Differentiate between a sample and a population. b. Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias.	7.SP.1a Differentiate between a sample and population.	7.SP.1b Understand that a sample is only part of a population.	7.SP.1c Recognize that everyone in an area (such as classroom) is a population.	<ul style="list-style-type: none"> Population can consist of people, animals, or objects. Population would be represented by a number. Population will be defined. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with items to be counted in a population.
<i>Broaden understanding of statistical problem solving.</i>				
7.SP.2 Broaden statistical reasoning by using the GAISE model: a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. <i>For example, "How do the heights of seventh graders compare to the heights of eighth graders?"</i> (GAISE Model, step 1) b. Collect Data: Design and use a plan to collect appropriate data to	7.SP.2a1 Formulate statistical questions that include simple comparisons. AND 7.SP.2a2 Collect data to answer statistical questions.	7.SP.2b Given two questions, identify which question is statistical (anticipates variability). For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because of the variability in students'	7.SP.2c Identify the 4 steps of the GAISE model.	<ul style="list-style-type: none"> Understand ordering. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with a Gaise model for example cut apart and manipulate the 4 steps.


				Learning Progression Building the Base & Engagement	
Learning Standard	Complexity a	Complexity b	Complexity c		
Most Complex					Least Complex
answer a statistical question. (GAISE Model, step 2) c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) d. Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4)		ages. (GAISE Model, step 1).			
Summarize and describe distributions representing one population and draw informal comparisons between two populations.					
7.SP.3 Describe and analyze distributions. a. Summarize quantitative data sets in relation to their context by using mean absolute deviation (MAD), interpreting mean as a balance point. b. Informally assess the degree of visual overlap of two numerical data distributions with roughly equal variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. <i>For example, the mean height of players on the basketball team is 10 cm greater than the mean</i>	7.SP.3a Answer simple questions given two data displays (e.g., which data set has more people?).	7.SP.3b Find the mean and median of a data set. Limit to 7 data points.	7.SP.3c Answer questions given a graph (e.g., given a histogram of student's heights, which range of heights did most students fall into?).	<ul style="list-style-type: none">Identify information on a graph which can include labels, scales, and data.Count data in a graph.Interpret data from a given graph.Recognize the symbols for addition (+), subtraction, (–), and equals (=) and identify the keys on a calculator.Solve “how many more” and “how many less” problems using information presented in the graphs.Recognize the symbols for addition (+), subtraction, (–), and equals (=) and identify the keys on a calculator.Identify what a key is on a graph.Read and interpret a key.	

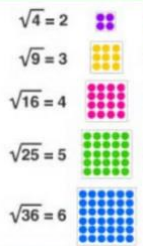
				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
<i>height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot (line plot), the separation between the two distributions of heights is noticeable.</i>				Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with a graph. Interact with a calculator.
7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event; a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely; and a probability near 1 indicates a likely event.	7.SP.5a Given an outcome in a real-life event or situation, such as a game, determine if an event is impossible, likely, unlikely, or certain.	7.SP.5b Given an outcome in a real-life event or situation, such as a game, determine if an event is impossible, likely, or unlikely.	7.SP.5c Given an outcome in a real-life event or situation, such as a game, determine if an event is possible or impossible.	<ul style="list-style-type: none"> Understand the term possible is something that can happen. Understand the term impossible is something that cannot occur. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Engage with probability manipulatives.
7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. <i>For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.</i>	7.SP.6a Approximate the probability of an event occurring as likely, unlikely, certain, or impossible based on possible outcomes using a model.	7.SP.6b Find the experimental probability of an event occurring after collecting data using a model.	7.SP.6c Collect data on the probability of an event (e.g. rolling dice, spinning a spinner, or drawing marbles).	<ul style="list-style-type: none"> Identify the possible outcomes. Understand methods to collect data. Understand methods to record data. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Engage with probability manipulatives.
7.SP.7 Develop a probability model and use it to find	7.SP.7a Compare the probabilities of an	7.SP.7b Use a probability	7.SP.7c Make prediction of the	<ul style="list-style-type: none"> Identify the possible outcomes. Understand methods to collect data.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
<p>probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.</p> <p>a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. <i>For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.</i></p> <p>b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?</p>	<p>event occurring (e.g., probability of landing on heads when flipping a coin; likelihood of landing on a certain area on a three section spinner).</p>	<p>model/graphic organizer to record data from a probability experiment (e.g., occurrence of heads or tails in a coin flip).</p>	<p>probability of an event occurring (e.g., probability of landing on heads when flipping a coin) using models.</p>	<ul style="list-style-type: none"> Understand methods to record data. Understand the term possible is something that can happen. Understand the term impossible is something that cannot occur. Count the number of times each outcome occurs. Recognize the outcome with the highest/lowest occurrence. Collect data from probability games and real world situations. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with probability manipulatives. Interact with probability games.
<p>7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations.</p> <p>a. Understand that, just as with simple events, the probability of a</p>	<p>7.SP.8a Find the number of outcomes of a compound events using a simple tree diagram, organized list, or</p>	<p>7.SP.8b Complete a simple tree diagram, organized list, or table (see example</p>	<p>7.SP.8c Find the probability of a simple event (e.g., probability of landing on</p>	<ul style="list-style-type: none"> Identify the possible outcomes. Understand methods to collect data. Understand methods to record data. Understand the term possible is something that can happen.

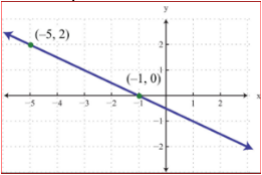
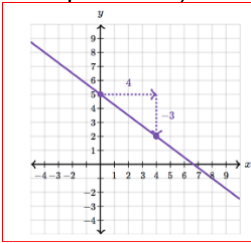
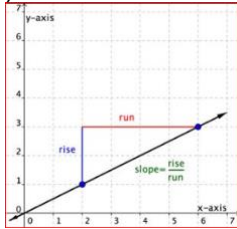
				Learning Progression <i>Building the Base & Engagement</i>	
Learning Standard	Complexity a	Complexity b	Complexity c		
Most Complex ←				→ Least Complex	
<p>compound event is the fraction of outcomes in the sample space for which the compound event occurs.</p> <p>b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language, e.g., “rolling double sixes,” identify the outcomes in the sample space which composes the event.</p> <p>c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least four donors to find one with type A blood?</p>	<p>table (see example below).</p>  <pre> graph LR SC[sugar cone] --- C1[chocolate] SC --- C2[vanilla] SC --- C3[strawberry] SC --- C4[mint] WC[waffle cone] --- C5[chocolate] WC --- C6[vanilla] WC --- C7[strawberry] WC --- C8[mint] </pre>	<p>blow).</p>  <pre> graph LR SC[sugar cone] --- C1[chocolate] SC --- C2[vanilla] SC --- C3[strawberry] SC --- C4[mint] WC[waffle cone] --- C5[chocolate] WC --- C6[vanilla] WC --- C7[strawberry] WC --- C8[mint] </pre>	<p>heads when flipping a coin).</p>	<ul style="list-style-type: none"> Understand the term impossible is something that cannot occur. Count the number of times each outcome occurs. Recognize the outcome with the highest/lowest occurrence. Collect data from probability games and real world situations. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with probability manipulatives. Interact with probability games. 	

Grade 8

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
The Number System				
<i>Know that there are numbers that are not rational, and approximate them by rational numbers.</i>				
8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating.	8.NS.1a Identify whether numbers are rational or irrational numbers.	8.NS.1b Identify whether numbers in decimal form are repeating or non-repeating decimals.	8.NS.1c Identify whether numbers are in the form of whole numbers, fractions or decimals.	<ul style="list-style-type: none"> Recognize that a number in whole number form has no fraction bar or decimal point. Recognize a number in decimal form has a point. Recognize that a number in fraction number form is two numbers separated by a line. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with fraction models.
8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., π^2 . <i>For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.</i>	8.NS.2a Estimate which point on a number line a decimal (up to hundredths) is closest to (e.g., given a number line in increments of $\frac{1}{10}$, identify which point the decimal 4.13 would be closest to).	8.NS.2b Round decimals to the nearest whole number or tenths and identify the corresponding points on a number line.	8.NS.2c Identify the whole number points on a number line.	<ul style="list-style-type: none"> Recognize numbers. Recognize a number line. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a number line.
Expressions and Equations				


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
<i>Work with radicals and integer exponents.</i>				
8.EE.1 Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. <i>For example, $3^2 \times 3^{-5} = 3^{(-3)} = 1/3^3 = 1/27$.</i>	8.EE.1a Apply properties of integer exponents to generate equivalent numerical expressions (e.g., $4^2 \times 4^3 = (4 \times 4) \times (4 \times 4 \times 4) = 4^5$).	8.EE.1b Identify equivalent numerical expressions with integer exponents; limit exponents to 1–6 (e.g., $3^4 = 3 \times 3 \times 3 \times 3$).	8.EE.1c Identify equivalent numerical expressions with integer exponents—limit to exponents 1–3 (e.g., $10^2 = 10 \times 10$).	<ul style="list-style-type: none"> Identify the base and exponent. Understand the exponent indicates the number of factors to be multiplied. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with numbers for the example of 5^4 give students a pile of cards with 5's and 4's written on. Have students select the number and types of cards to represent 5^4.
8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that it is irrational.	8.EE.2a Construct a perfect square up to 25 (e.g., 5 squared is 25). 	8.EE.2b Create a representation of a perfect square.	8.EE.2c Select the perfect square, given a model.	<ul style="list-style-type: none"> Identify a number represented by array (columns and rows). Demonstrate knowledge of a square and a rectangle. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with number models.
8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as	8.EE.3a Identify equivalent expressions of numbers expressed in the form of a single	8.EE.3b Identify equivalent expressions of multiples of 10 using exponents (limit	8.EE.3c Identify equivalent expressions of multiples of 10 using exponents	<ul style="list-style-type: none"> Identify the base and exponent. Understand the exponent indicates the number of factors to be multiplied.

				Learning Progression <i>Building the Base & Engagement</i>
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Most Complex ←————→ Least Complex				
much one is than the other. <i>For example, estimate the population of the United States as 3×10^8; and the population of the world as 7×10^9; and determine that the world population is more than 20 times larger.</i>	digit times a whole number power of 10 (limit exponent to 1–10) (e.g., What is 3×10^3 ? Answer: 3000 or $3 \times 10 \times 10 \times 10$).	exponent to 1–10) (e.g., What is 10^7 ? Answer: $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$).	(limit exponent to 1–5) (e.g., What is 10^2 ? Answer: 10×10).	Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with numbers for the example of 10^2 give students a pile of cards with 10's and 2's written on. Have students select the number and types of cards to represent 10^2.
8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology.	8.EE.4a Given a real-world context, write a number in scientific notation that best represents the situation.	8.EE.4b Given a real-world context and a selection of numbers written in scientific notation, select the quantity that best represents the situation.	8.EE.4c Interpret scientific notation that has been generated by technology.	<ul style="list-style-type: none"> Identify relevant buttons on a calculator to scientific notation. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with calculator.
<i>Understand the connections between proportional relationships, lines, and linear equations.</i>				
8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. <i>For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</i>	8.EE.5a Identify the slope (unit rate) of a line of a proportional graph represented on a grid that has scales of 1.	8.EE.5b Graph a simple proportion with 3 coordinates.	8.EE.5c Identify if a graph represents a proportional relationship.	<ul style="list-style-type: none"> Identify points on a horizontal number line (scale limited to whole numbers 1-10). Identify points on a vertical number line (scale limited to whole numbers 1-10). Understand a coordinate grid is formed by a vertical and horizontal number line. Recognize the horizontal number line as the x-axis. Recognize the vertical number line as the y-axis.

				Learning Progression <i>Building the Base & Engagement</i>
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Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Recognize the intersection of the x-axis and y-axis as the origin. Identify a line. Identify the point (0,0) on a graph. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a graph.
8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .	8.EE.6a Identify the slope of a line using a graph (see example below). 	8.EE.6b Identify the slope of a line using a graph in the first quadrant (see example below). 	8.EE.6c Identify the slope of a line using a graph in the first quadrant when the rise and run are shown on the graph (see example below). 	<ul style="list-style-type: none"> Discuss slope represented in every-day life (roof, stairs, ramp, hill, ski slope, road, incline plane, etc.) Identify an x, y axis graph (visually or tactually displayed) Identify that a graph can show slope Identify the x and y axis (horizontal and vertical line) Associate slope with graphed triangles Identify rise and run. Identify that slope is written as a fraction. Identify that the rise is the number on top and run is the number on the bottom. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a graph.
<i>Analyze and solve linear equations and pairs of simultaneous linear equations.</i>				

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
8.EE.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	8.EE.7a Solve a 1-step linear equation (e.g., $y + 3 = 5$).	8.EE.7b Identify the operation needed to solve a given 1-step linear equation (the inverse operation).	8.EE.7c When given a visual model, the students will correctly identify the missing variable from given choices (select from no more than 3, e.g., given the equation $4 + x = 2$, identify that the variable is x).	<ul style="list-style-type: none"> Know that a variable can represent an unknown value. Identify a number sentence. Recognize that the signs and numbers are not variables. Identify numbers. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with no more than 3 answer choices be able to select 1 from different positions. Interact with a visual model.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
8.EE.8 Analyze and solve pairs of simultaneous linear equations graphically. a. Understand that the solution to a pair of linear equations in two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously. b. Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. <i>For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.</i> c. Solve real-world and mathematical problems leading to pairs of linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (Limit solutions to those that can be addressed by graphing.)	8.EE.8a Identify the coordinate at which two lines intersect.	8.EE.8b Locate the point where two lines intersect.	8.EE.8c Determine whether two lines intersect.	<ul style="list-style-type: none"> Know that a line is not curved. Know that when two lines are the same distance apart will not intersect. Know that intersect means the lines will cross. Know that just because you do not see two lines intersect does not mean they do not. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with real-world examples of lines.
Functions				
<i>Define, evaluate, and compare functions.</i>				

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in grade 8.	8.F.1a Determine if a relation is a function when given in table form.	8.F.1b Determine if a relation is a function when given in graph form.	8.F.1c Identify the inputs and outputs of a function given in table form.	<ul style="list-style-type: none"> Know the meaning of the words <i>input</i> and <i>output</i>. Relate <i>input</i> and <i>output</i> to a function machine. Understand that a table is made up of columns and rows. Know that input is in the left column and the output is in right column. Columns are up and down. Rows are left and right. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with tables.
8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</i>	8.F.2a Compare functions represented in the same form.	8.F.2b Classify graphs of functions as linear or non-linear. 	8.F.2c Determine whether a line is increasing, decreasing, or flat (zero slope).	<ul style="list-style-type: none"> Know what a line is. Know you need to read a graph from left to right. Know that flat means not going up or down. Know that increasing is up and decreasing is down. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with real-world examples of lines.
8.F.3 Interpret the equation $y = mx + b$ as defining a linear function,	8.F.3a Match a function to its graph.	8.F.3b Determine whether a function is	8.F.3c Determine whether the slope	<ul style="list-style-type: none"> Identify points on a horizontal number line (scale limited to whole numbers 1-10).

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
<p>whose graph is a straight line; give examples of functions that are not linear. <i>For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.</i></p>		<p>linear or non-linear given the equation or graph.</p>	<p>of the function is positive, negative or zero.</p>	<ul style="list-style-type: none"> Identify points on a vertical number line (scale limited to whole numbers 1-10). Understand a coordinate grid is formed by a vertical and horizontal number line. Recognize the horizontal number line as the x-axis. Recognize the vertical number line as the y-axis. Recognize the intersection of the x-axis and y-axis as the origin. Identify a line. Represent a numeral with physical objects. Count physical objects up to 100 by ones and tens. Demonstrate understanding of the word "equal". Recognize that in real-world increasing is adding quantities. For example, buying more objects increases the price. Connecting these situations with the vocabulary of "positive slope". Recognize that in real-world decreasing is removing quantities. For example, buying more objects decreases the amount of money you have. Connecting these situations with the vocabulary of "negative slope". Recognize that in real-world there are situations where there is no change in the dependent variable. For example, an adult height does not change over time. Connecting these situations with the vocabulary of "zero slope".


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Understand that a graph is read from left to right. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with real-world examples of lines. Interacting with physical objects and understanding when you add objects you are increasing the amount and when you remove objects you are decreasing the amount.
<i>Use functions to model relationships between quantities.</i>				
8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	8.F.4a Graph a linear function on a grid with a scale of 1.	8.F.4b Given a graph representing a linear equation, identify the slope and y-intercept.	8.F.4c Identify two points on a linear graph.	<ul style="list-style-type: none"> Identify points on a horizontal number line (scale limited to whole numbers 1-10). Identify points on a vertical number line (scale limited to whole numbers 1-10). Understand a coordinate grid is formed by a vertical and horizontal number line. Recognize the horizontal number line as the x-axis. Recognize the vertical number line as the y-axis. Recognize the intersection of the x-axis and y-axis as the origin. Identify a line. Identify the point (0,0) on a graph. Represent a numeral with physical objects. Count the number of objects in a given group (bananas and oranges).

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Identify the first number in an ordered pair as the x coordinate (movement in horizontal direction from the origin). Identify the second number in an ordered pair as the y coordinate (movement in vertical direction from the origin). Recognize that a line is made up of many points. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with real-world examples represented by a line.
8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	8.F.5a Tell a story using the qualitative features of a function.	8.F.5b Identify if a function in graph form is linear or nonlinear.	8.F.5c Identify if a function in graph form is increasing or decreasing or flat.	<ul style="list-style-type: none"> Recognize that in real-world increasing is adding quantities. For example, buying more objects increases the price. Connecting these situations with the vocabulary of “positive slope”. Recognize that in real-world decreasing is removing quantities. For example, buying more objects decreases the amount of money you have. Connecting these situations with the vocabulary of “negative slope”. Recognize that in real-world there are situations where there is no change in the dependent variable. For example, an adult height does not change over time. Connecting these situations with the vocabulary of “zero slope”.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Understand that a graph is read from left to right. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with real-world examples represented by a line.
Geometry				
<i>Understand congruence and similarity using physical models, transparencies, or geometry software.</i>				
8.G.1 Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). a. Lines are taken to lines, and line segments are taken to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	8.G.1a Show experimentally (e.g., by measuring, overlapping figures, etc.) that congruent shapes have the same angle measures and side lengths.	8.G.1b Identify corresponding parts (angles and sides) on congruent shapes.	8.G.1c Identify congruent line segments and/or congruent angles.	<ul style="list-style-type: none"> Same size and shape or equal in measure should be referred to as congruent. Identify the same sized angles in a simple 2D shape. Identify lengths of sides in a simple 2D shape. Recognize there are different types of triangles and quadrilaterals. Identify a line segment and an angle. Identify angles and side lengths in a simple 2D shape. Recognize that the size of an angle depends on the size of an opening not the length of the rays. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with different length real-world objects, e.g., straws, pencils, pens, by

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				rotating one of the objects from the other. Realize that the length of the “rays” does not make a difference, angles can still be the same size.
8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them (include examples both with and without coordinates).	8.G.2a Determine the sequence of transformations (rotation, reflection, translation) that will make a figure congruent to another limit to two transformations in the sequence.	8.G.2b Determine whether a rotation, a reflection or a translation is needed to show whether one figure is congruent to another (limit to 1 transformation).	8.G.2c Determine the direction and how many units a figure must be translated (shifted) to be congruent to another on a coordinate plane (e.g., 3 units to the right).	<ul style="list-style-type: none"> Recognize that a translation is a slide. Recognize that a slide can be a horizontal movement or vertical movement, a vertical movement, or a combination of both. Count units on a grid. Counting units on a grid recognizing that you count spaces between the gridlines and not the intersections of the gridlines. Understand that congruence means same size and shape or equal in measure. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with 2D objects on a plane moving them in straight direction until they overlap.
8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	8.G.3a Compare the effects of dilations, translations, rotations and reflections (with or without coordinate grids). And/or perform a translation, rotations, and reflection on a grid.	8.G.3b Recognize the effect of rotation (turn), reflection (flip), and translation (slide).	8.G.3c Demonstrate concepts of translation (up, down, right, left) with manipulatives.	<ul style="list-style-type: none"> Recognize that a translation is a slide. Recognize that a slide can be a horizontal movement or vertical movement, a vertical movement, or a combination of both. <p>Engagement Statements (demonstration of engaged in the topic)</p>

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Interact with 2D objects on a plane moving them in straight direction.
8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them (include examples both with and without coordinates).	8.G.4a Perform a dilation on a figure using a scale factor of $\frac{1}{2}$, 2, or 3.	8.G.4b Recognize the effects of dilation on a two-dimensional figure (with or without coordinate grids). For example, recognize that applying a scale factor greater than one creates a bigger image and a scale factor less than one creates a smaller image.	8.G.4c Recognize and identify similar shapes and congruent figures (with or without coordinate grids). For example, if given a sheet of pictures of turtles or stars, recognize that the stretched images are not similar.	<ul style="list-style-type: none"> Understand that similar means the same shape but different size. <p>means same size and shape or equal in measure. ed equally in both directions with technology. ed in one direction with technology. r images.</p> <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with objects (Shrinky Dinks) and/or technology that can show similarity in stretching.
8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. <i>For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.</i>	8.G.5a Given a pair of parallel lines cut by a transversal, identify corresponding angles, alternate interior angles, alternate exterior angles, supplementary angles, and vertical angles.	8.G.5b Know the sum of the interior angles of a triangle equals 180 degrees.	8.G.5c Identify triangles, parallel lines, perpendicular lines, and intersecting lines.	<ul style="list-style-type: none"> Recognize that a triangle has 3 angles and 3 sides. Know that when two lines are the same distance apart they will not intersect. Know that intersect means the lines will cross. Know that just because you do not see two lines intersect does not mean they do not. Know that perpendicular lines are a special type of intersecting lines that form 90 degree angles. Recognize right angle notation.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
				<ul style="list-style-type: none"> Recognize right angle (square corner) is 90 degrees. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with real-world examples of lines. Interact with real world examples of triangles
<i>Understand and apply the Pythagorean Theorem.</i>				
8.G.6 Analyze and justify an informal proof of the Pythagorean Theorem and its converse.	8.G.6a Know that triangles with the side lengths of 3, 4, 5, 6, 8, and 10 are right triangles.	8.G.6b Identify a right triangle when drawn on a coordinate plane.	8.G.6c Identify right triangles from a group of a variety of triangles.	<ul style="list-style-type: none"> Recognize that a triangle has 3 angles and 3 sides. Recognize right angle notation. Recognize right angle (square corner) is 90 degrees. Recognize that there are different types of triangles. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with real world examples of all types of triangles.
8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	8.G.7a Find the length of the hypotenuse of a right triangle when given the lengths of the legs.	8.G.7b Place the numbers into the Pythagorean Theorem when given legs and	8.G.7c Identify parts of a right triangle (hypotenuse, legs).	<ul style="list-style-type: none"> Recognize that the longest side of a right triangle is opposite the right (square) angle. Identify right triangles shown in different orientations. Identify the sides of any triangle.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
		hypotenuse of a right triangle.		<ul style="list-style-type: none"> Identify the length of the sides of triangles. Differentiate between a side and the side length measure of a side of a triangle. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with real world examples of all types of triangles.
8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	8.G.8a Find the length of vertical and horizontal lines drawn on the coordinate grid.	8.G.8b Identify vertical and horizontal lines of a triangle drawn on the coordinate grid.	8.G.8c Identify vertices of a triangle on the coordinate grid.	<ul style="list-style-type: none"> Recognize that a triangle has 3 angles and 3 sides. Recognize that the pointy part of a shape is a vertex. Recognize that a shape has more than one vertex. Understand a coordinate grid is formed by a vertical and horizontal number line. Recognize the horizontal number line as the x-axis. Recognize the vertical number line as the y-axis. Identify a line segment. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with real world examples of all types of triangles.
<i>Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.</i>				

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
8.G.9 Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres.	8.G.9a Solve real-world and mathematical problems involving volume of cylinders.	8.G.9b Match the given formula for volume to cones, cylinders and spheres.	8.G.9c Find cones, cylinders, and spheres in the environment.	<ul style="list-style-type: none"> Recognize that a cone has a circular base and a comes to a point at the opposite end. Recognize that a cylinder has two congruent circular bases. Recognize that a sphere is shaped like a ball. Recognize that cylinders and spheres have no pointed ends. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with objects shaped like cylinders, cones and spheres.
Statistics and Probability				
<i>Investigate patterns of association in bivariate data.</i>				
8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association. (GAISE Model, steps 3 and 4)	8.SP.1a Construct a scatter plot for bivariate data using no more than 10 data points.	8.SP.1b Identify if a scatter plot has linear or nonlinear association.	8.SP.1c Identify if the pattern for a scatter plot is increasing or decreasing.	<ul style="list-style-type: none"> Points on a scatter plot represent data. The points on a scatter plot are not connected. Recognize that in real-world increasing is adding quantities. Recognize that in real-world decreasing is removing quantities. Recognize that in real-world there are situations where there is no change in the dependent variable. Understand that a graph is read from left to right

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
8.SP.2 Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (GAISE Model, steps 3 and 4)	8.SP.2a Determine which line most closely represents the line of best fit for a given scatterplot.	8.SP.2b Determine whether patterns on a scatter plot are positive, negative, or have no correlation.	8.SP.2c Determine whether a linear graph is increasing, decreasing, or flat.	<ul style="list-style-type: none"> Recognize a linear graph. Recognize that in real-world increasing is adding quantities. For example, buying more objects increases the price. Connecting these situations with the vocabulary of “positive slope”. Recognize that in real-world decreasing is removing quantities. For example, buying more objects decreases the amount of money you have. Connecting these situations with the vocabulary of “negative slope”. Recognize that in real-world there are situations where there is no change in the dependent variable. For example, an adult height does not change over time. Connecting these situations with the vocabulary of “zero slope”. Understand that a graph is read from left to right. Recognize a coordinate plane. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interacting with physical objects and understanding when you add objects you are increasing the amount and when you remove objects you are decreasing the amount. Interact with graphs, both increasing and decreasing.


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. <i>For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.</i> (GAISE Model, steps 3 and 4)	8.SP.3a Given a line of best fit, make a prediction.	8.SP.3b Given several lines on a scatterplot, identify which line most closely represents the line of best fit.	8.SP.3c Determine whether the line of best fit should have a positive, negative, or zero slope.	<ul style="list-style-type: none"> Recognize a linear graph. Recognize that in real-world increasing is adding quantities. For example, buying more objects increases the price. Connecting these situations with the vocabulary of “positive slope”. Recognize that in real-world decreasing is removing quantities. For example, buying more objects decreases the amount of money you have. Connecting these situations with the vocabulary of “negative slope”. Understand that a graph is read from left to right. Recognize a coordinate plane. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact using technology with e.g., images people of same age and different length.
8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible	8.SP.4a Identify an inside missing value in a table of values using the pattern.	8.SP.4b Find the totals in a two-way frequency table given the sums (outside values in the table).	8.SP.4c Identify the total population in a two-way frequency table (lower right-hand box).	<ul style="list-style-type: none"> Understand what total means Count objects. Understand where the numbers come from in a simple two-way table, e.g., using the students in the class as the total, split in “Boys and Girls” and “In Choir or Other” <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a table.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
association between the two variables. <i>For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?</i>				<ul style="list-style-type: none"> Interact with data.

Extended Standards with Learning Progressions for High School

*Additional mathematics standards for High School that represent complex numbers on the complex plane in rectangular and polar form (including real and Imaginary numbers) and are indicated by a + symbol in the Ohio's New Learning Standards are not included in the extended standards since they are not considered common mathematics curriculum for all college and career ready students.

Numbers and Quantity

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
The Real Number System				
<i>Extend the properties of exponents to rational exponents.</i>				
N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.</i>	N.RN.1a Identify equivalent expression with exponents (e.g. $3 \times 3 \times 3$ is 3^4).	N.RN.1b Identify equivalent expression with exponents (limit to squares) (e.g. 3×3 is 3^2).	N.RN.1c Identify a number with an exponent.	This is in Algebra II
N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	N.RN.2a Identify equivalent expression with exponents and radicals (e.g. $3 \times 3 \times 3$ is 3^{3^4}).	N.RN.2b Identify equivalent expression with exponents and radicals (e.g. 3×3 is 3^2).	N.RN.2c Identify a number with a radical.	This is in Algebra II
<i>Use properties of rational and irrational numbers.</i>				

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	N.RN.3a Recognize the effects of multiplying and dividing with negative numbers (e.g., $-2 \times -4 = 8$).	N.RN.3b Recognize that the absolute value of a rational number is how far it is from 0 on the number line (e.g., plot a number and its opposite on a number line and recognize that they are equidistant from zero).	N.RN.3c Recognize that addition means move to the right and subtraction means move to the left on a number line.	This is in Algebra II
Quantities				
<i>Reason quantitatively and use units to solve problems.</i>				
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	N.Q.1a Solve real-world problems involving positive and negative numbers (e.g., temperatures, elevations, and distance from a fixed point (map reading)).	N.Q.1b Solve problems involving positive and negative numbers using a number line (e.g., temperatures, distances from a fixed point).	N.Q.1c Locate a given positive or negative number on a number line.	<ul style="list-style-type: none"> • Demonstrate or act out a situation • Identify units in a given situation. • Recognize units of measure. • Show what addition and subtraction represents. • Use technology to add and subtract unit measures. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> • Engage with measuring quantities. • Interact with concrete objects. • Interact with a graph or a chart.


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.	N.Q.2a Identify the appropriate unit of measure for volume.	N.Q.2b Identify the appropriate unit of measure for length.	N.Q.2c Identify units of measure.	<ul style="list-style-type: none"> • Demonstrate or act out a situation • Recognize Interact with technology.units of measure <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> • Engage with measuring quantities • Interact with concrete objects • Interact with situations involving units of measure
N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	N.Q.3a Identify the appropriate unit of measure for volume.	N.Q.3b Identify the appropriate unit of measure for length.	N.Q.3c Identify units of measure.	<ul style="list-style-type: none"> • Use technology to calculate unit measures. • Demonstrate or act out a situation. • Recognize units of measure. • Understand what a whole number is. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> • Engage with measuring quantities. • Interact with concrete objects. • Interact with situations involving units of measure. • Interact with technology.
The Complex Number System				
<i>Perform arithmetic operations with complex numbers.</i>				

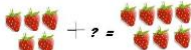
				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a+bi$ with a and b real.	N.CN.1a Describe real and complex numbers.	N.CN.1b Given a set of numbers, compare the real and complex numbers.	N.CN.1c Given a set of numbers, label the real and complex numbers.	Not on BP
N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	N.CN.2a Add and subtract complex numbers.	N.CN.2b Identify a complex number.	N.CN.2c Identify an imaginary number.	Not on BP
<i>Use complex numbers in polynomial identities and equations.</i>				
N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.	N.CN.7a Solve a quadratic equation with real solutions.	N.CN.7b Given a graph of a quadratic equation, match the correct equation.	N.CN.7c Identify a coefficient.	Not on BP

Algebra

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
Seeing Structure in Expressions				
<i>Interpret the structure of expressions.</i>				
A.SSE.1. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	A.SSE.1a Represent a real-world situation with an expression, both numerals and variables. Recognize parts of the expression in the real-world situation.	A.SSE.1b Represent a real-world situation with a numeric expression. Recognize parts of the expression in the real-world situation.	A.SSE.1c Represent a real-world situation with a model using concrete objects.	<ul style="list-style-type: none"> Demonstrate or act out the situation Show what addition, subtraction, multiplication and division represents using manipulatives. Recognize the symbols for addition (+), subtraction (−), multiplication (×), division (÷), and equals (=). <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with concrete objects. Interact with models.
A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the " $x - 5$ " is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.	A.SSE.2a Simplify expressions involving variables (e.g., $(2(x + 4) = 2x + 8)$).	A.SSE.2b Identify the equivalent numeric expression (e.g., $7 + 5 = 5 + 7$).	A.SSE.2c Identify equivalent expressions with whole numbers less than 10 using concrete objects (e.g., objects, dots, etc.).	Not on BP
<i>Write expressions in equivalent forms to solve problems.</i>				


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. (A1, M2) b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (A1, M2) c. Use the properties of exponents to transform expressions for exponential functions. <i>For example, 8^t can be written as 2^{3t}.</i>	A.SSE.3a Apply properties of integer exponents to generate equivalent variable expressions (e.g., $b^2 \times b^4 = b^6$).	A.SSE.3b Apply properties of integer exponents to generate equivalent numerical expressions (e.g., $5^2 \times 5^4 = 5^6$).	A.SSE.3c Interpret numerical expressions with exponents (e.g., 5^4 means $5 \times 5 \times 5 \times 5$).	<ul style="list-style-type: none"> • Demonstrate or act out the situation. • Show what addition, subtraction, multiplication and division represents using manipulatives. • Identify the base and the exponent of an exponential expression. • Recognize the symbols for addition (+), subtraction (−), multiplication (×), division (÷), and equals (=). • Count physical objects up to 30. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> • Interact with concrete objects. • Interact with models and pictures. • Interact with representations of the unknown.
Arithmetic with Polynomials and Rational Expression Standards				
<i>Perform arithmetic operations on polynomials.</i>				
A.APR.1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. a. Focus on polynomial expressions that simplify to forms	A.APR.1a Add and subtract linear and/or quadratic polynomials. Models may be used.	A.APR.1b Add and subtract linear polynomials. Models may be used.	A.APR.1c Add linear polynomials. Models may be used.	Not on BP


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
that are linear or quadratic. (A1, M2) b. Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic. (A2, M3)				
<i>Understand the relationship between zeros and factors of polynomials.</i>				
A.APR.2 Understand and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$. In particular, $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	A.APR.2a Multiply two binomials.	A.APR.2b Multiply a variable by a binomial.	A.APR.2c Identify a polynomial (binomials only).	Not on BP
A.APR.3 Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.	A.APR.3a Find the zeros of a polynomial when the polynomial is factored (e.g., $x^2 - 9 = 0$ and $x^2 = 3x = 2 = 0$).	A.APR.3b Identify a polynomial (trinomial) (e.g., $x^2 = 3x = 2 = 0$).	A.APR.3c Identify a polynomial (binomial) (e.g., $x^2 - 9 = 0$).	Not on BP
<i>Rewrite rational expressions.</i>				
A.APR.6 Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more	A.APR.6a Identify a rational expression (e.g., $\frac{6x}{3} = 2x$).	A.APR.6b Rewrite expressions in different forms (e.g., $x^2 + 1 = (x * x) + 1$).	A.APR.6c Given a visual model, identify an expression (e.g. $2 * 2 * 2 = 2^3$).	Not on BP

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
complicated examples, a computer algebra system.				
Creating Equations Standards				
<i>Create equations that describe numbers or relationships.</i>				
A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. a. Focus on applying linear and simple exponential expressions. (A1, M1) b. Focus on applying simple quadratic expressions. (A1, M2) c. Extend to include more complicated function situations with the option to solve with technology. (A2, M3)	A.CED.1a Represent and solve a real-world situation with a two-step linear equation or inequality (e.g., Abby has \$15 to spend on a snack and two matching T-shirts. If she spends \$3 on the snack, what is the maximum price of each T-shirt? Key: $x \leq 5$).	A.CED.1b Represent and solve a real-world problem with a one-step linear equation or inequality (e.g., Abby has \$5, and she wants to buy a T-shirt for \$8. How much more money does she need? Key: $5 + x = 8$).	A.CED.1c Represent a real-world problem with a linear equation, using concrete objects, models and pictures (see example below). 	<ul style="list-style-type: none"> Demonstrate or act out the situation. Recognize the unknown. Show what addition, subtraction, multiplication and division represents using manipulatives. Know that a symbol \square or letter can represent a missing value. Count physical objects up to 30. Relate a picture or objects to a number sentence. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with concrete objects. Represent numbers. Interact with representations of the unknown.
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	A.CED.2a Create an equation with two variables to represent a linear relationship between quantities in a given context (e.g., $y = 2x + 4$).	A.CED.2b Using a two-variable equation describing a real-world situation, given the value of one variable, find and interpret the value of the other	A.CED.2c Identify the meaning of each number and/or variable in a given two-variable equation	<ul style="list-style-type: none"> Recognize variables. Recognize that one variable affects the other variable. Recognize that a variable can represent any number. Demonstrate that a coefficient represents a constant decrease or increase.


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
<p>a. Focus on applying linear and simple exponential expressions. (A1, M1)</p> <p>b. Focus on applying simple quadratic expressions. (A1, M2)</p> <p>c. Extend to include more complicated function situations with the option to graph with technology. (A2, M3)</p>		<p>variable (e.g., Sally starts with \$4 and gets an allowance of \$2 each week. After x weeks, she has $y = 2x + 4$ dollars. When $x = 3$, find y and interpret the result).</p>	<p>that describe a real-world situation (e.g., Sally starts with \$4 and gets an allowance of \$2 each week. After x weeks she has $y = 2x + 4$ dollars. What does 4 represent? What does x represent?).</p>	<ul style="list-style-type: none"> Demonstrate or act out the situation. Recognize the unknown. Show what addition, subtraction, multiplication and division represents using manipulatives. Know that a symbol \square or letter can represent a missing value. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with concrete objects. Represent quantities. Interact with representations of the unknown.
<p>A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i> (A1, M1)</p> <p>a. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)</p>	<p>A.CED.3a Represent a constraint with an equation or inequality in two variables (e.g., $x + y \leq 8$, describing the number of boys and girls in an 8-passenger van).</p>	<p>A.CED.3b Create a one-variable constraint using an inequality (e.g., $x \leq 6$).</p>	<p>A.CED.3c Demonstrate a constraint using words or models (e.g., how many students can fit at this table?).</p>	<p>[Between c and b:</p> <ul style="list-style-type: none"> Identify the symbols $<$, $>$, and $=$.] Recognize the unknown. Demonstrate quantities using manipulatives. Understand the order of numbers. Identify limitations of a real-world situation. Compare quantities using manipulatives or pictures to identify which is more and which is less. <p>Engagement Statements (demonstration of engaged in the topic)</p>


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> • Demonstrate or act out a situation. • Interact with real-world situations with restraints, e.g., seats at a table, passengers in a bus, eggs in a carton. • Interact with situations involving greater than, equal to, or less than. • Interact with a model of a real-world situation.
A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. a. Focus on formulas in which the variable of interest is linear or square. For example, rearrange Ohm's law $V = IR$ to highlight resistance R , or rearrange the formula for the area of a circle $A = (\pi)r^2$ to highlight radius r . (A1) b. Focus on formulas in which the variable of interest is linear. For example, rearrange Ohm's law $V = IR$ to highlight resistance R . (M1) c. Focus on formulas in which the variable of interest is linear or square. For example, rearrange the formula for the area of a circle $A = (\pi)r^2$ to highlight radius r . (M2) d. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)	A.CED.4a Rearrange a one-step formula to highlight a quantity (e.g., use the formula $a=lw$ to highlight the length of the rectangle by rearranging it to $l=a/w$).	A.CED.4b Rearrange a one-step equation to solve for a variable (e.g., solve for x : $y=2+x$, $y/2=x$).	A.CED.4c Match a formula to a given situation (e.g., recognize that $a=lw$ is the formula for the area of a rectangle)	<ul style="list-style-type: none"> • Recognize that a formula represents a situation. • Recognize what the variables represent in the formula. • Model a situation with manipulatives. • Relate a picture or objects to a number sentence. • Read and interpret a traditional one-step number sentence ($2 \times 3 = \square$). • Identify a number sentence. • Recognize the symbols for addition (+), subtraction (−), multiplication (×), division (÷), and equals (=). • Recognize a numerical expression with and without variables. • Demonstrate or act out a situation. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> • Interact with real-world situations.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
				<ul style="list-style-type: none"> Interact with a model of a real-world situation. Interact with representations of the unknown.
Reasoning with Equations and Inequalities Standards				
<i>Understand solving equations as a process of reasoning and explain the reasoning.</i>				
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	A.REI.1a Order a given sequence of steps to solve an equation (e.g., $2x + 5 = 13$). Solve two-step equations with integer coefficients and solutions, explaining the steps.	A.REI.1b Determine a step needed to solve a two-step equation.	A.REI.1c Determine the step needed to solve a one-step equation (e.g., to solve $x + 5 = 13$, subtract 5 from both sides).	[Between C and B: <ul style="list-style-type: none"> Identify the first step in solving a two-step problem involving addition and subtraction.] Demonstrate the meaning of the solution. Understand that the variable represents the unknown. Understand that the operations addition and subtraction are inverse operations. Understand the the operations multiplication and divisions are inverse operations. Understand that the equal sign is a balance. Understand that an operation applied to one side needs to be applied to the other. Use manipulatives to understand the situation. Demonstrate or act out a situation.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
				Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> • Interact with manipulatives that represent a situation. • Interact with real-world situations. • Interact with a model of a real-world situation. • Interact with representations of the unknown. • Interact with a balance scale.
A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	A.REI.2a Solve linear equations with more than one step.	A.REI.2b Solve 1-step linear equations.	A.REI.2c Solve for the missing number within a given number sentence involving addition or subtraction of numbers less than 10.	Not on BP
<i>Solve equations and inequalities in one variable.</i>				
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	A.REI.3a Solve a two- or three-step linear equation in one variable. Models may be used.	A.REI.3b Solve a one-step linear equation in one variable. Models may be used.	A.REI.3c Given a linear equation in one-variable and a list of possible solutions, identify the solution of the equation.	<ul style="list-style-type: none"> • Understand the idea of what a solution to an equation means. • Identify the solution of the equation using manipulatives. • Using trial error explore the idea of the equal sign being a balance. • Identify a number sentence.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Recognize the symbols for addition (+), subtraction (−), multiplication (×), division (÷), and equals (=). Read and interpret a traditional one-step number sentence ($2 \times 3 = \square$). Relate a picture or objects to a number sentence. Know that a symbol \square or letter can represent a missing value. Count to 30. Count physical objects up to 30. Recognize a numerical expression with and without variables. Identify a numerical expression without exponents. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a balance scale. Interact with physical objects (blocks) or drawings that represent an expression or an equation. Interact with physical objects (blocks) or drawings representing addition, subtraction, or multiplication word problems.
A.REI.4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an	A.REI.4a Identify or create perfect squares (e.g., square root of $25 = 5$).	A.REI.4b Identify equivalent expressions that are cubes (e.g., $m^3 = m \times m \times m$).	A.REI.4c Identify equivalent expressions that are squared (e.g., $m^2 = m \times m$).	Not on BP

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
equation of the form $(x - p)^2 = q$ that has the same solutions. b. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.				
<i>Solve systems of equations.</i>				
A.REI.6 Solve systems of linear equations algebraically and graphically. a. Limit to pairs of linear equations in two variables. (A1, M1) b. Extend to include solving systems of linear equations in three variables, but only algebraically. (A2, M3)	A.REI.6a Identify the coordinate at which two lines intersect.	A.REI.6b Locate the point on the graph at which two lines intersect.	A.REI.6c Identify whether two lines intersect.	<ul style="list-style-type: none"> Identify a line. Identify crossing lines in the real-world. Show that the length of intersecting lines can be extended so they will eventually cross. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with manipulatives representing lines.
A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.</i>	A.REI.7a Locate the coordinate of the point(s) at which a line intersects a quadratic function (e.g., at which two coordinates does the	A.REI.7b Locate the point(s) on the graph at which a line intersects a quadratic function (e.g., identify on the graph where the line intersects the parabola).	A.REI.7c Identify whether a line intersects a quadratic function (e.g., does the line intersect the parabola at one or two points?	Not on BP

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
	line intersect the parabola?).		Does the line intersect the parabola?).	
<i>Represent and solve equations and inequalities graphically.</i>				
A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	A.REI.10a Given a graph and an equation, fill out three points on a corresponding table of values.	A.REI.10b Given a table of values, graph the line on the coordinate plane.	A.REI.10 Identify a point on a line on a coordinate plane.	<ul style="list-style-type: none"> • Locate an ordered pair (x, y) as a point on the coordinate plane. • Plot a point on the coordinate plane. • Identify the x- and y- axis. • Identify that the x- and y- axes are number lines. • Identify a point. • Demonstrate that on a number line and on the coordinate plane the spaces need to be counted not the grid lines. • Identify a line. • Locate the origin at the point (0, 0) on the coordinate plane. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> • Interact with the coordinate grid.
A.REI.11 Explain why the x-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, making tables of values,	A.REI.11a Locate the coordinate at which two lines intersect. Using the x coordinate of the intersection point, substitute it back into the original equation	A.REI.11b Locate the coordinate point on the graph at which two lines intersect.	A.REI.11c Identify whether two lines intersect.	<ul style="list-style-type: none"> • Identify a line. • Identify crossing lines in the real-world. • Show that non-parallel lines can be extended so they eventually cross.


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
or finding successive approximations).	to show that it is a solution of the equation.			Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with the coordinate grid. Interact with manipulatives representing lines.
A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	A.REI.12a Given a graph of an inequality including the shaded region, identify three points that make the inequality true.	A.REI.12b Identify on a graph of a line \leq , \geq is represented by a solid line; and $<$ and $>$ are represented by a dotted line.	A.REI.12c Identify the graph of a linear inequality has a shaded region.	<ul style="list-style-type: none"> Identify ordered pairs that satisfy an inequality. Plot ordered pairs that satisfy an inequality. Understand that a linear inequality has more than one solution. Understand that the solutions to a linear equation in two variables are all the points on a straight line. Understand that the solutions to a linear inequality in two variables are points on a plane. Each point on a coordinate plane is associated to a (x,y) ordered pair. Using trial error explore ordered pairs that satisfy an inequality Using trial error explore ordered pairs that satisfy an equation. Identify a shaded region on a coordinate plane. Identify a shaded region in a picture or of an object. Engagement Statements (demonstration of engaged in the topic)

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none">• Interact with a graph• Interact with the coordinate grid.

Functions


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
Interpreting Functions Standards				
<i>Understand the concept of a function, and use function notation.</i>				
F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.	F.IF.1a Determine if a relation is a function.	F.IF.1b Complete an input-output table when given the function rule and values.	F.IF.1c Identify the input or output of a function given in table form.	<p>[Between b and c:</p> <ul style="list-style-type: none"> Interact with real-world situations that can be represented by functions. Interact with relations that are not functions (e.g., Talk about pets, making Dog, Cat, and Horse as the input, ask the question "Which student has which pet?" and establish the correspondence, to decide if the relation is or is not a function.) Establish the ordered pairs between the corresponding elements of the input and output.] Show real world relations in table form. Know the meaning of the words <i>input</i> and <i>output</i>. Relate <i>input</i> and <i>output</i> to a function machine. Understand that a table is made up of columns and rows. Know that input is in the left column and the output is in right column. Columns are up and down. Rows are left and right.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> • Interact with constructing tables. • Interact with real-world situations. • Interact with a function machine. • Interact with technology.
F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	F.IF.2a Given a linear equation using function notation, complete a table of values.	F.IF.2b Represent an equation in $y=$ form with $f(x)$.	F.IF.2c Understand that $f(x)=y$.	<ul style="list-style-type: none"> • Experience real world concepts that are represented by a symbol. (e.g., street signs, symbols on technology tools, etc.) • Recognize that a function can be written as abstract mathematical symbolic language • Levels F.IF.1a-c need to be mastered. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> • Interact with constructing tables. • Interact with real-world situations. • Interact with a function machine. • Interact with technology.
F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. <i>For example, the Fibonacci sequence is defined recursively by $f(0) = f(1)$</i>	F.IF.3a Given a sequence, determine the functional rule.	F.IF.3b Predict the next three terms in an arithmetic or geometric sequence (e.g., 3,6,9 ...).	F.IF.3c Identify the common ratio or common difference in a sequence.	<ul style="list-style-type: none"> • Describe the “rule” for a pattern. • Identify the “rule” for a pattern. • Discover patterns in the real world • Build or draw patterns using technology or manipulatives. • Translate patterns to numbers

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
$= 1, f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.				<ul style="list-style-type: none"> Replicate a given pattern. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with patterns
<i>Interpret functions that arise in applications in terms of the context.</i>				
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (A2, M3) a. Focus on linear and exponential functions. (M1) b. Focus on linear, quadratic, and exponential functions. (A1, M2)	F.IF.4a Given a function made up of several linear functions, determine where the function is increasing, decreasing, or flat	F.IF.4b Given a graph of a linear equation, identify the y and/or x intercept.	F.IF.4c Determine whether the linear function is increasing, decreasing, or flat.	<ul style="list-style-type: none"> Identify the x- and y- axis. Read the graph from left to right. Recognize that the x- and y- axes are number lines. Recognize a line. Recognize patterns of the line going up or down, or staying at the same level, reading from left to right. Experience the creation of graphs using science probes Demonstrate stories of a given graph. Create a graph to a story using technology. Know that flat means not going up or down. Know that increasing is up and decreasing is down. Recognize that in real-world increasing is adding quantities. For example, buying more objects increases the price. Connecting these situations with the vocabulary of “positive slope”. Recognize that in real-world decreasing is removing quantities. For example, buying

				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex				Least Complex
				<p>more objects decreases the amount of money you have. Connecting these situations with the vocabulary of “negative slope”.</p> <ul style="list-style-type: none"> Recognize that in real-world there are situations where there is no change in the dependent variable. For example, an adult height does not change over time. Connecting these situations with the vocabulary of “zero slope”. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with real-world examples of lines.
<p>F.IF.5 Relate the domain of a function to its graph, and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</p> <p>a. Focus on linear and exponential functions. (M1)</p> <p>b. Focus on linear, quadratic, and exponential functions. (A1, M2)</p> <p>c. Emphasize the selection of a type of function for a model based</p>	<p>F.IF.5a Given the graph represented by a linear function, determine the domain.</p>	<p>F.IF.5b Given a context of a linear equation, describe the domain.</p>	<p>F.IF.5c Given a table, state the input values.</p>	<ul style="list-style-type: none"> Given a real world situation, e.g., When I buy 1 chocolate bar - I pay 2 dollars, when I buy 2 chocolate bars - I pay 4 dollars, match the values in the situation with the values in the table. Count physical objects. Relate the input and the output to a function machine. Answer questions about a table. Identify what the input is in a table. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with real-world situations that can be represented by functions.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
on behavior of data and context. (A2, M3)				<ul style="list-style-type: none"> Interact with constructing tables. Interact with relations that are functions. Explore a chart or a table.
F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (A2, M3)	F.IF.6a Identify the slope of a line when the equation is written in slope intercept form.	F.IF.6b Identify the slope of a line when given in graph form.	F.IF.6c Determine whether a slope is present on a given visual graph.	Not on BP
<i>Analyze functions using different representations.</i>				
F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. a. Graph linear functions and indicate intercepts. (A1, M1) b. Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2) c. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (A2, M3) d. Graph polynomial functions, identifying zeros, when factoring is	F.IF.7a1 Graph a linear function using a graph with a scale of 1. AND/OR F.IF.7a2 Determine whether an ordered pair is a viable solution to a given linear function.	F.IF.7b1 Determine the y intercept point for a linear graph. AND/OR F.IF.7b2 Determine whether the line is increasing (going up), decreasing (going down), or flat.	F.IF.7c1 Identify two point on a linear graph. AND/OR F.IF.7c2 Classify graphs of functions as linear or non-linear.	<ul style="list-style-type: none"> Identify the x- and y- axis. Recognize that the x- and y- axes are number lines. Recognize a line vs. a curve. Draw lines and curves. Use technology to explore graphing. Recognize where the lines intersect the axes <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with graphs. Interact with technology.


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex				Least Complex
reasonable, and indicating end behavior. (A2, M3) e. Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1) f. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (A2, M3)				
<i>Analyze functions using different representations.</i>				
F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3) i. Focus on completing the square to quadratic functions with the leading coefficient of 1. (A1, M2) b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change ^G in functions such as $y = (1.02)^t$, and $y = (0.97)^t$ and classify them as representing exponential growth or decay. (A2, M3)	F.IF.8a Identify equivalent expressions (e.g. $2x + 2x = 4x$, $x^2 \cdot x^2 = x^4$).	F.IF.8b Identify equivalent expressions (limit to three terms) (e.g. $x + x + x = 3x$, $x \cdot x \cdot x = x^3$).	F.IF.8c Identify equivalent expressions (limit to two terms) (e.g. $x + x = 2x$).	Not on BP


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
i. Focus on exponential functions evaluated at integer inputs. (A1, M2)				
F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i> (A2, M3) a. Focus on linear and exponential functions. (M1) b. Focus on linear, quadratic, and exponential functions. (A1, M2)	F.IF.9a Compare a function given in table form to another function given in graphical form. <i>For example, which one is increasing?</i>	F.IF.9b Match a function given in table form to its graph.	F.IF.9c Match a function given as a verbal description to its graph. <i>For example, which is an increasing function?</i>	<ul style="list-style-type: none"> Demonstrate stories to match an increasing/decreasing graph. Match an increasing/decreasing graph to a story. Connect to F.IF.4 and F.IF.3 Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with graphs. Interact with technology. Experience and act out real world situation that can be graphed (e.g., distance/time graphs).
Building Functions Standards				
<i>Build a function that models a relationship between two quantities.</i>				
F.BF.1 Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from context. i. Focus on linear and exponential functions. (A1, M1)	F.BF.1a Create a linear function that represents a linear relationship between quantities in a given context.	F.BF.1b Given a linear function that describes a real-world situation and given the value of one variable, find and interpret the value of the other variable.	F.BF.1c Identify the meaning of each number and/or variable in a linear function that describes a real-world situation.	<ul style="list-style-type: none"> Count physical objects. Relate the input and the output to a function machine. Answer questions about a table. Identify what the input is in a table. Interpret variables and numbers in the linear function that describes the real-world situation


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
<p>ii. Focus on situations that exhibit quadratic or exponential relationships. (A1, M2)</p> <p>b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (A2, M3)</p> <p>c. Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.</p>				<ul style="list-style-type: none"> Know that a variable can represent an unknown value. Identify a number sentence. Recognize that the signs and numbers are not variables. Identify numbers. Relate a picture or objects to a number sentence. Identify a number sentence. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with no more than 3 answer choices be able to select 1 from different positions. Interact with a visual model. Interact with real-world situations described by linear functions Interact with a model of a real-world situation. Interact with representations of the unknown.
<p>F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula. Use them to model situations and translate between the two forms.</p>	<p>F.BF.2a Identify the rule for a pattern.</p>	<p>F.BF.2b Identify the next term in a pattern.</p>	<p>F.BF.2c Determine if a given set represents a pattern.</p>	<ul style="list-style-type: none"> Experience patterns and non-patterns. Discover patterns in the real world. Build patterns using manipulatives. Draw the patterns. Translate the patterns to numbers. Recognize the informal “rule” for the pattern.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
				Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Engage with patterns using manipulatives
<i>Build new functions from existing functions.</i>				
F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3) a. Focus on transformations of graphs of quadratic functions, except for $f(kx)$. (A1, M2)	F.BF.3a Identify a line reflected over the y-axis on a coordinate grid.	F.BF.3b Identify a line reflected over the x-axis on a coordinate grid.	F.BF.3c Identify a line on a coordinate grid.	Not on BP
F.BF.4 Find inverse functions. a. Informally determine the input of a function when the output is known. (A1, M1)	F.BF.4a Solve for x when y is given (e.g. $y = x+3$; what is the value of x when y is 5?).	F.BF.4b Identify the input and output of a function.	F.BF.4c Identify a function.	Not on BP
Linear Quadratic and Exponential Models Standards				
<i>Construct and compare linear, quadratic, and exponential models, and solve problems.</i>				


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	F.LE.1a Identify a situation that represents a linear and/or exponential function.	F.LE.1b Identify the graph of a linear function and an exponential function.	F.LE.1c Identify a graph of a linear function.	<ul style="list-style-type: none"> Sort graphs based on their shape. Draw lines and curves Recognize a line vs. a curve Observe graphs of different functions with technology <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with technology Interact with no more than 3 answer choices be able to select 1 from different positions. Interact with a visual model.
F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).	F.LE.2a After creating a sequence, make a graph that represents that sequence.	F.LE.2b Create a geometric sequence of at least 5 numbers with models.	F.LE.2c Create an arithmetic sequence with a model.	<ul style="list-style-type: none"> Translate patterns to numbers Build patterns using manipulatives Draw the patterns Recognize the informal “rule” for the pattern Experience patterns and non-patterns. Discover patterns in the real world. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Engage with patterns using manipulatives Interact with models of patterns.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex 				Least Complex
F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. (A1, M2)	F.LE.3a Observe a situation that shows increasing and decreasing linear and exponential events. (e.g., doubling a penny every day gives you more money than receiving \$100 a day for a month)	F.LE.3b Identify an exponential function.	F.LE.3c Identify if a linear function is increasing or decreasing.	Not on BP
F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct}=d$ where a , c , and d are numbers, and the base b is 2, 10, or e ; evaluate the logarithm using technology.	F.LE.4a Identify equivalent expressions with exponents.	F.LE.4b Identify equivalent expressions with exponents (limit to power 3 expressions) (e.g. Which is the same as $m \times m \times m$?).	F.LE.4c Identify equivalent expressions with exponents (limit to power 2 expressions) (e.g. Which is the same as $m \times m$?).	Not on BP
<i>Interpret expressions for functions in terms of the situation they model.</i>				
F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.	F.LE.5a Given a context, interpret the parameters of an exponential function (e.g., during x weeks, the existing number of fish in the pond has been doubled). This situation is modeled by the exponential	F.LE.5b Given a context, interpret the parameters of a linear function (e.g., Marsha has \$10 already saved and saves an additional \$5 a week for x number of weeks). This situation is modeled by a linear function $f(x) = 5x + 10$,	F.LE.5c Identify the constant in a linear or exponential equation. OR When given the graph of a function, identify the y intercept.	Not on BP

				Learning Progression Building the Base & Engagement	
Learning Standard	Complexity a	Complexity b	Complexity c		
Most Complex					Least Complex
	function $y = 100(2^x)$, where 100 is the initial number of fish in the pond, 2 is a growth factor, (2^x) is the number by which the initial number of fish, 100, is multiplied for every increase in x , and y is the total number of fish in the pond.	where 10 is the initial amount that has been saved, 5 is the weekly saving, $5x$ is the amount of money saved during x weeks, and $f(x)$ is a total amount of money saved including the initial amount.			
Trigonometric Functions Standards					
Extend the domain of trigonometric functions using the unit circle.					
F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	F.TF.1a Identify the measure of a central angle on a circle when the measure of the arc is given.	F.TF.1b Identify the measure of an angle.	F.TF.1c Identify an angle.	Not on BP	
F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	F.TF.2a Identify the measure of a central angle on a circle when the measure of the arc is given.	F.TF.2b Identify the measure of an angle.	F.TF.2c Identify an angle.	Not on BP	
Model periodic phenomena with trigonometric functions.					


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex 				Least Complex
F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	F.TF.5a Identify the measure of a central angle on a circle when the measure of the arc is given.	F.TF.5b Identify the measure of an angle.	F.TF.5c Identify an angle.	Not on BP
<i>Prove and apply trigonometric identities.</i>				
F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	F.TF.8a Find the hypotenuse when the length of the sides is given.	F.TF.8b Identify the parts of a right triangle.	F.TF.8c Identify a right triangle.	Not on BP

Geometry


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
Congruence				
<i>Experiment with transformations in the plane.</i>				
G.CO.1 Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length.	G.CO.1a Identify points, lines, line segments, angles (right, acute, obtuse, and order by size), and perpendicular and parallel lines.	G.CO.1b Identify points, lines, line segments and angles (right, acute, obtuse, and order by size).	G.CO.1c Identify points, lines, and line segments, and order angles by size.	<ul style="list-style-type: none"> Understand that an angle is created by two (straight) rays that meet at a point Draw geometric figures (e.g. points, lines, line segments and angles) Recognize different angles in real - world situations Use manipulatives to create angles and recognize that the size of the angle is related to the amount of rotation of the initial ray. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with geometric figures (e.g. points, lines, rays, line segments and angles). Interact with line segments and angles in the real world.
G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare	G.CO.2a Demonstrate that a rotation (turn), a reflection (flip), or a translation (slide) maps a figure onto another.	G.CO.2b Identify whether a rotation (turn), a reflection (flip), or a translation (slide) can map a figure onto another.	G.CO.2c Match shapes in different orientations. (i.e., shapes = 2D)	[Between levels b and c: <ul style="list-style-type: none"> Recognize the orientation of objects using terms such as above, below, in front of, behind, and next to.]

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Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.				<ul style="list-style-type: none"> Match drawings of shapes having the same orientation. Match shapes using manipulatives. Recognize objects in the environment using names of shapes. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a variety of shapes (e.g. using pattern blocks)
<p>G.CO.3 Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself.</p> <p>a. Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes.</p> <p>b. Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes.</p>	G.CO.3a Show that two figures have symmetry on a coordinate plane.	G.CO.3b Identify figures that have line symmetry or rotational symmetry, using concrete objects or on a coordinate plane.	G.CO.3c Given visual models, determine which figures have line symmetry. (i.e., figure =3D)	<ul style="list-style-type: none"> Sort photos of real - world shapes with and without line symmetry. Identify shapes without line symmetry. Explore the concept of line symmetry in the real - world. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with shapes. Interact with visual models of shapes with line symmetry. Observe a demonstration of shape folding, using shapes with and without line symmetry.
G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles,	G.CO.4a Identify that a translation requires a direction	G.CO.4b Identify whether a transformed figure is	G.CO.4c Identify whether a transformed figure	<ul style="list-style-type: none"> Match drawings of figures having the same and different orientation.

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Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
circles, perpendicular lines, parallel lines, and line segments.	and distance; a rotation requires a center and angle; and a reflection requires a line.	a “translation,” “reflection,” or “rotation.”	is a “slide,” “flip,” or “turn.”	<ul style="list-style-type: none"> Experience different rigid transformations in combination with the vocabulary. Match the figures' orientation with the type of the transformation. Demonstrate with hand movement or technology what the terms, slide, flip, and turn mean. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a variety of 2D - shapes (e.g. using pattern blocks) including angles and line segments. Observe a demonstration of transformations using technology and manipulatives.
G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	G.CO.5a Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure.	G.CO.5b Given visuals or real-world items, demonstrate a rotation (turn), a reflection (flip), or a translation (slide).	G.CO.5c Match shapes in different orientations.	<p>[Between levels b and c:</p> <ul style="list-style-type: none"> Recognize the orientation of shapes using terms such as slide, flip, and turn.] Match drawings of shapes having the same orientation. Match shapes using manipulatives. Recognize shapes in the environment using the names of shapes. Experience different rigid transformations in combination with the vocabulary.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex				Least Complex
				<ul style="list-style-type: none"> Match the shapes' orientation with the type of the transformation <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a variety of shapes (e.g. using pattern blocks)
<i>Understand congruence in terms of rigid motions.</i>				
G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent	G.CO.6a Identify a basic rigid motion (a rotation (turn), a reflection (flip), or a translation (slide) that maps one figure onto another. (Restrict to situations in which a single basic rigid motion suffices.)	G.CO.6b Show two figures are congruent by demonstrating that a rotation (turn), a reflection (flip), or a translation (slide) maps one onto the other.	G.CO.6c Match shapes to show congruence by placing one figure on top of the other.	<ul style="list-style-type: none"> Recognize congruent shapes. Experience different rigid transformations Match the shapes' orientation with the type of the transformation, in combination with the vocabulary, flip, slide, and turn. Recognize when two similar shapes are the same size or not. Recognize shapes that are the same. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a variety of 2D - shapes, including angles and line segments Observe demonstrations of shapes mapping onto each other or not, using technology or manipulatives.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	G.CO.7a Identify whether a rotation (turn), a reflection (flip), or a translation (slide) is required to show that a triangle is congruent to another triangle on a coordinate plane. Limit to one transformation.	G.CO.7b Identify whether a rotation (turn), a reflection (flip), or a translation (slide) is required to show that a triangle is congruent to another triangle. Limit to one transformation.	G.CO.7c Match triangles in different orientations.	[Between levels c and b: <ul style="list-style-type: none"> Recognize congruent triangles.] Experience different rigid transformations. Match the triangles' orientation with the type of the transformation, in combination with the vocabulary, flip, slide, and turn. Match drawings of triangles having the same orientations. Recognize when two similar triangles are the same size or not. Recognize triangles that are the same shape. Identify triangles. G.CO.10 needs to be taught before G.CO.7 Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with a variety of triangles, including angles and line segments Observe demonstrations of triangles mapping onto each other or not, using technology or manipulatives.
G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition	G.CO.8a Determine whether two triangles are	G.CO.8b Match corresponding parts (sides and angles) of congruent triangles.	G.CO.8c Match one corresponding part (side or angle)	<ul style="list-style-type: none"> Given two congruent triangles with different orientations, identify a corresponding angle or side.


				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex 				Least Complex
of congruence in terms of rigid motions.	congruent using ASA, SAS, or SSS.		of two congruent triangles.	<ul style="list-style-type: none"> Recognize corresponding parts - sides and angles Recognize angles and sides of a triangle. Understand that an angle in a triangle is created by two sides that meet at a point (vertex) G.CO.10 needs to be taught first. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a variety of congruent triangles. Observe demonstrations of triangles mapping onto each other or not, using technology or manipulatives.
Prove geometric theorems both formally and informally using a variety of methods.				

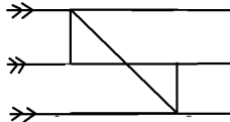
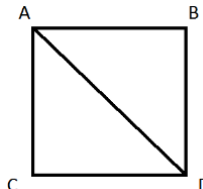
				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
G.CO.9 Prove and apply theorems about lines and angles. Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	G.CO.9a1 Identify a pair of vertical, complementary, supplementary, corresponding, alternative interior, or alternate exterior angles. G.CO.9a2 Find a missing angle measure for situations involving vertical, complementary, supplementary, corresponding, alternative interior, and alternative exterior angles.	G.CO.9b1 Given a pair of vertical angles and a missing angle measurement, find the missing angle measure. G.CO.9b2 Bisect a line segment using a ruler, compass, technology, or other means and label the midpoint. G.CO.9b3 Create a pair of perpendicular lines using a ruler, compass, technology or other means. Include the right-angle marking.	G.CO.9c1 Identify vertical angles. G.CO.9c2 Identify a set of perpendicular lines. G.CO.9c3 Identify the midpoint of a line segment. Identify a right angle.	<ul style="list-style-type: none"> Recognize or draw intersecting lines. Sort intersecting lines by angles that meet perpendicular or not. Among intersecting lines recognize the special case of perpendicular lines. Discover vertical angles. Discover right angles/square corners. Understand that angles are created by two lines that meet at a point. Recognize angles formed by intersecting lines. Understand that the midpoint is equal distance from each end point. Manipulate a drawn line segment, drawn on e.g., patty paper, to recognize the midpoint. Identify a line segment. Identify a line. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a variety of intersecting lines.
G.CO.10 Prove and apply theorems about triangles. Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180° ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle	G.CO.10a Determine the sum of the measures of the interior angles of a triangle. Identify congruent angles in isosceles and equilateral triangles.	G.CO.10b Identify right, equilateral, and isosceles triangles.	G.CO.10c Identify a triangle.	<ul style="list-style-type: none"> Understand that a triangle is a closed shape. Understanding a triangle has three points (vertices) and three sides. Sort a variety of shapes to recognize triangles. Count the number of sides of a shape.


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
is parallel to the third side and half the length; the medians of a triangle meet at a point.				<ul style="list-style-type: none"> Count the number of angles of a shape. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with a variety of shapes to recognize triangles.
G.CO.11 Prove and apply theorems about parallelograms. Theorems include but are not restricted to the following: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	G.CO.11a Identify the congruent sides and angles of a parallelogram.	G.CO.11b Identify congruent sides on a parallelogram.	G.CO.11c Identify a rectangle and a parallelogram.	<ul style="list-style-type: none"> Sort quadrilaterals based on the presence or absence of right angles. Discover right angles/square corners. Understand that a quadrilateral is a closed shape. Sort a variety of figures to recognize quadrilaterals. Sort quadrilaterals and describe their rule. Count the number of sides of a shape. Count the number of angles of a shape. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with a variety of shapes to recognize quadrilaterals.
<i>Make geometric constructions.</i>				
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective	G.CO.12a Construct a circle given a center and a radius. Copy a segment.	G.CO.12b Construct a line segment given its endpoints.	G.CO.12c Identify geometric tools (e.g., straightedge, protractor, and	<ul style="list-style-type: none"> Match tools with drawings and their names.

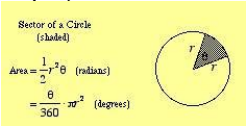
				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.			ruler) and their uses.	<ul style="list-style-type: none"> Observe the usage of geometric tools, e.g., straight edge, ruler, protractor, and compass. Explore/use geometric tools. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with geometric tools (e.g., straightedge, protractor, compass, and ruler)
G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	G.CO.13a Construct an equilateral triangle.	G.CO.13b Construct a circle given a center and point on the circle.	G.CO.13c Given three congruent line segments (or sticks), make an equilateral triangle.	<ul style="list-style-type: none"> Understand “congruent triangles” as being of equal side- length and angle measures. Interact with geometric tools (e.g., straightedge, protractor, and ruler). Recognize and identify triangles with equal side lengths. Sort triangles by equal or different side lengths. Sort triangles by equal or different angle measures. Use manipulatives to create triangles. Recognize triangles in the real-world. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with triangles in the real-world.
Classify and analyze geometric figures.				


				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex ←————→ Least Complex				
G.CO.14 Classify two-dimensional figures in a hierarchy based on properties.	G.CO.14a Classify two-dimensional shapes based on their properties.	G.CO.14b Sort different types of quadrilaterals.	G.CO.14c Sort different types of triangles.	<ul style="list-style-type: none"> Recognize and identify triangles with equal side lengths. Sort triangles by the presence or absence of a right angle, an angle larger than a right angle, or three angles smaller than a right angle. Sort triangles by the number of equal side lengths. Sort triangles by the number of equal angle measures. Use manipulatives to create triangles. Recognize triangles in the real-world. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with triangles in the real-world. Interact with and sort a variety of shapes to recognize triangles.
Similarity Right Triangles and Trigonometry				
<i>Use complex numbers in polynomial identities and equations.</i>				
G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves	G.SRT.1a Determine the dimensions of a figure after dilation.	G.SRT.1b Determine if a figure is bigger or smaller after dilation.	G.SRT.1c Compare 2 figures to determine if a dilation has occurred.	Not on BP

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
a line passing through the center unchanged.				
G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations, the meaning of similarity for triangles as the equality of all corresponding pairs of angles, and the proportionality of all corresponding pairs of sides.	G.SRT.2a Determine if figures are similar; describe or select why two figures are or are not similar.	G.SRT.2b Determine if two rectangles or triangles are similar.	G.SRT.2c Identify similar triangles.	Not on BP
G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	G.SRT.3a Identify similar triangles in different orientations.	G.SRT.3b Identify similar triangles.	G.SRT.3c Identify a triangle.	Not on BP
<i>Prove and apply theorems both formally and informally involving similarity using a variety of methods.</i>				
G.SRT.4 Prove and apply theorems about triangles. Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean theorem proved using triangle similarity.	G.SRT.4a Identify different types of triangles.	G.SRT.4b Identify parts of a right triangle.	G.SRT.4c Identify a right angle in the environment.	Not on BP
G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures	G.SRT.5a Identify if triangles are similar, or not, in a given geometric figure;	G.SRT.5b Identify if triangles are similar or not in a	G.SRT.5c Identify similar triangles.	Not on BP


				Learning Progression Building the Base & Engagement	
Learning Standard	Complexity a	Complexity b	Complexity c		
Most Complex				Least Complex	
that can be decomposed into triangles.	e.g. 	decomposed polygon; e.g., is triangle ABD similar to triangle DCA? 			
Define trigonometric ratios, and solve problems involving right triangles.					
G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	G.SRT.6a Identify parts of a right triangle.	G.SRT.6b Identify right triangles.	G.SRT.6c Identify a triangle.	Not on BP	
G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.	G.SRT.7a Identify parts of a right triangle.	G.SRT.7b Identify right triangles.	G.SRT.7c Identify an angle of a triangle.	Not on BP	
G.SRT.8 Solve problems involving right triangles. a. Use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. (G, M2) b. Use trigonometric ratios and the Pythagorean theorem to solve right	G.SRT.8a Construct a right triangle on a coordinate plane and label the parts.	G.SRT.8b Identify the parts of a right triangle (right angle, legs, and hypotenuse).	G.SRT.8c Given an assortment of triangles, identify right triangles.	Not on BP	


				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
triangles in applied problems. (A2, M3)				
Circles				
<i>Understand and apply theorems about circles.</i>				
G.C.1 Prove that all circles are similar using transformational arguments.	G.C.1a Compare two circles and determine how to change one to make it the same as the other (e.g., circle A needs to be enlarged to match circle B).	G.C.1b Label the parts of a circle.	G.C.1c Locate circles in the environment.	Not on BP
G.C.2 Identify and describe relationships among angles, radii, chords, tangents, and arcs, and use them to solve problems. Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	G.C.2a Use the radius of a circle to determine the length of the diameter and vice versa.	G.C.2b Identify parts of a circle (radius, diameter, circumference, chord, and arc).	G.C.2c Locate circles in the environment.	Between level c and b: <ul style="list-style-type: none"> Recognize the difference between the circle (the points equal distance from the midpoint) and the area of the circle. Recognize circles in a collection of different shapes. Recognize that a circle is not a sphere. Know that a circle has no straight lines. Know that a circle has no vertices/corners). Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with a variety of shapes.

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
G.C.3 Construct the inscribed and circumscribed circles of a triangle, prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle.	G.C.3a Identify a circumscribed circle about a triangle.	G.C.3b Identify a circle inscribed in a triangle.	G.C.3c Identify a circle.	BP??
<i>Find arc lengths and areas of sectors of circles.</i>				
G.C.4 Find arc lengths and areas of sectors of circles. a. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. b. Derive the formula for the area of a sector, and use it to solve problems.	G.C.4a Identify the central angle of a circle. OR Apply the formula to the area of a sector (e.g., area of a slice of pie). 	G.C.4b Identify the sector of a circle.	G.C.4c Identify the arc of a circle.	Not on BP
G.C.5 Derive formulas that relate degrees and radians, and convert between the two. (A2, M3)	G.C.5a Identify the central angle of a circle.	G.C.5b Identify the sector of a circle.	G.C.5c Identify the arc of a circle.	Not on BP
Expressing Geometric Properties with Equations				
<i>Translate between the geometric description and the equation for a conic section.</i>				
G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean theorem; complete the square to find the	G.GPE.1a Identify the diameter of a circle.	G.GPE.1b Identify the radius of a circle.	G.GPE.1c Identify a circle.	Not on BP

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex				Least Complex
center and radius of a circle given by an equation.				
<i>Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.</i>				
G.GPE.4 Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. <i>For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle.</i> (G, M2)	G.GPE.4a Find the perimeter of quadrilaterals drawn on a coordinate grid.	G.GPE.4b Identify shapes on a coordinate grid.	G.GPE.4c Identify special triangles, quadrilaterals, and circles.	Not on BP
G.GPE.5 Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.	G.GPE.5a Describe the “rise and run” relationships between two perpendicular lines.	G.GPE.5b Identify the slopes of parallel and perpendicular lines in a coordinate grid.	G.GPE.5c Identify parallel and perpendicular lines in a coordinate grid.	<ul style="list-style-type: none"> • Use manipulatives to create parallel and perpendicular lines. • Identify parallel lines in the real-world. • Sort lines in to parallel and perpendicular. • Sort intersecting lines by angles that meet perpendicular or not. • Among intersecting lines recognize the special case of perpendicular lines. • Discover right angles/square corners. • Understand that angles are created by two lines that meet at a point. • Recognize angles formed by intersecting lines. • Identify a line.

				Learning Progression <i>Building the Base & Engagement</i>
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Most Complex ←————→ Least Complex				
				Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with manipulatives, eg., straws and skewers. Interact with a coordinate grid.
G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	G.GPE.6a Find the midpoint of a vertical or horizontal line on a coordinate grid.	G.GPE.6b Find the length of a vertical or horizontal line on a coordinate grid.	G.GPE.6c Identify points, lines, and line segments.	Not on BP
G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	G.GPE.7a Find the area and perimeter of shapes given on a coordinate grid. (Restrict to shapes with sides that are vertical or horizontal line segments.)	G.GPE.7b Find the perimeter of shapes given on a coordinate grid. (Restrict to shapes with sides that are vertical or horizontal line segments.)	G.GPE.7c Identify shapes on a coordinate grid.	<ul style="list-style-type: none"> Use manipulatives to create shapes. Identify shapes in the real-world. Sort shapes. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with manipulatives. Interact with a coordinate grid. Interact with physical objects that represent 2-D shapes in the real-world, e.g., the lid on a sandwich box, a window, etc.
Geometric Measurement and Dimension				
<i>Explain volume formulas, and use them to solve problems.</i>				

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex 				Least Complex
G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	G.GMD.1a Compare the volume of two objects with the same base but different heights and vice versa (e.g., Which cup can hold more water: the shorter or the taller cup; given the choice of different sized cubes, identify which would hold more).	G.GMD.1b Distinguish between objects that do and do not have volume.	G.GMD.1c Sort three-dimensional objects (cones, cylinders, spheres).	Not on BP
G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	G.GMD.3a Compare the volume of two objects with the same base but different heights and vice versa (e.g., Which cup can hold more water: the shorter or the taller cup; given the choice of different sized cubes, identify which would hold more).	G.GMD.3b Distinguish between objects that do and do not have volume.	G.GMD.3c Sort three-dimensional objects (cones, cylinders, spheres, pyramids).	Not on BP
<i>Visualize relationships between two-dimensional and three-dimensional objects.</i>				
G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and	G.GMD.4a Identify cross-sections of	G.GMD.4b Identify faces of three-dimensional shapes.	G.GMD.4c Identify two- and three-	Not on BP

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex  Least Complex				
identify three-dimensional objects generated by rotations of two-dimensional objects.	three- dimensional shapes.		dimensional shapes.	
<i>Understand the relationships between length, area, and volume.</i>				
G.GMD.5 Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures.	G.GMD.5a Compare the volume of three-dimensional shapes.	G.GMD.5b Compare the area of shapes.	G.GMD.5c Compare similar shapes.	Not on BP
G.GMD.6 When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k , the effect on lengths, areas, and volumes is that they are multiplied by k , k^2 , and k^3 , respectively.	G.GMD.6a Find the volume of two similar three-dimensional shapes.	G.GMD.6b Find the area of two similar shapes.	G.GMD.6c Identify similar shapes.	Not on BP
Modeling with Geometry				
<i>Apply geometric concepts in modeling situations.</i>				
G.MG.1 Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.	G.MG.1a Connect the shape of real-world objects to two-dimensional and three-dimensional shapes (e.g., the trunk of a tree is cylindrical in shape; a car is cube in shape; the center of a sunflower is circular in shape; a	G.MG.1b Connect the shape of real-world objects to two-dimensional shapes (e.g., a window is rectangular in shape, a wheel is circular in shape, and a table can be of many different shapes).	G.MG.1c Connect two-dimensional shapes with real-world objects.	Not on BP

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
	bookshelf is rectangular prism in shape).			
G.MG.2 Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.	G.MG.2a Calculate and compare the densities of two datasets in the same modeling situation (e.g, Is the population density of Ohio or New York greater?).	G.MG.2b Calculate the density of a given situation.	G.MG.2c Given representations of density, identify the one with the greatest or least density (e.g., If 3 squares of the same size have different numbers of dots in them, which one has the greatest number of dots?).	Not on BP
G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.	G.MG.3a Compare the volume of real-world objects.	G.MG.3b Compare the area of real-world objects.	G.MG.3c Sort shapes that model a real-world object (e.g., a baseball is a sphere, a can of soup is a cylinder).	Not on BP

Statistics and Probability

				Learning Progression <i>Building the Base & Engagement</i>
Learning Standard	Complexity a	Complexity b	Complexity c	
Most Complex ←————→ Least Complex				
Interpreting Categorical and Quantitative Data				
<i>Summarize, represent, and interpret data on a single count or measurement variable.</i>				
S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots) in the context of real-world applications using the GAISE model.	S.ID.1a Collect data in real-world context to create a dot plot, histogram, or box plot to represent collected data.	S.ID.1b Create a dot plot, histogram, or a box plot to represent given or collected data.	S.ID.1c Match given data to a given dot plot, histogram, or box plot.	<p>[Between level c and b:</p> <ul style="list-style-type: none"> Identify a missing whole number value on a number line marked with whole number up to 10.] Sort representations of different data displays. Recognize different data representations, table, dot plot, histogram and box plot. Gather data, e.g., the height of the people in the classroom (at least 11 heights) and organize the same data in table, histogram, and box plot. Recognize that on a number line the spaces need to be counted not the grid lines, assuming a scale of 1. Understand that 2 is the distance from 0 to 2 and 3 is the distance from 0 to 3 using standard units for all lengths from 1 to 10. Identify 0 on a number line. Recognize a point on a number line. Know what a number line is.

				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Know the order of the numbers from 0 to 10. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a variety of data representations, i.e. dot plot, histogram or box plot. Engage with graphing technology. Engage with a group of students to gather real-world data and observe the data being organized into different displays.
S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets.	S.ID.2a Compare mean, median, and mode of 2 or more given graphs or collected data sets.	S.ID.2b Compute mean, median, or mode of a given graph or collected data set involving numbers less than 100.	S.ID.2c Identify the median and mode of a graph or a given data set involving numbers less than 50.	<ul style="list-style-type: none"> Order a set of an odd quantity of numbers, e.g., 11 data points, from least to greatest. Gather real-world data points. Interact with arithmetic operations. Recognize the symbols for addition (+), subtraction (−), multiplication (×), division (÷), and equals (=). Count up to 50. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with technology Engage with a group of students to gather real-world data and observe the data being organized into different displays.


				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex ←————→ Least Complex				
S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	S.ID.3a Interpret a dot plot.	S.ID.3b Interpret a histogram.	S.ID.3c Complete an incomplete dot plot, box plot, or histogram (e.g., adding missing labels and missing data points).	<ul style="list-style-type: none"> S.ID.1b: Create a dot plot, histogram, or a box plot to represent given or collected data. Match the vocabulary to the correct display. Understand the labels on a data display. Recognize a point on a number line. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with a variety of data representations, i.e. dot plot, histogram or box plot Engage with a group of students to gather real-world data and observe the data being organized into different displays.
S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	S.ID.4a Organize given data into a normal distribution graph.	S.ID.4b Formulate the mean of given data for the normal distribution.	S.ID.4c Identify the normal distribution of given data.	Not on BP
<i>Summarize, represent, and interpret data on two categorical and quantitative variables.</i>				


				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex ←————→ Least Complex				
S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	S.ID.5a Determine the missing value in a two-way frequency table using the given context.	S.ID.5b Given a two-way frequency table, within a context, determine the missing value(s).	S.ID.5c Identify the “most” or “least” value in a two-way frequency table.	<p>[Between levels c and b:</p> <ul style="list-style-type: none"> • Enter the data numerically in the two - way table • Create a table with labels for columns and rows • Sort cards with data into 4 cells of the table • Gather the data on e.g. index cards. • Recognize a question that involves two categories (e.g. gender/music style, gender/pet ownership.)] • Relate “greater than” and “less than” to “most” and “least”. • Determine between two numbers which is “greater than” or “less than”. • Order ten different numbers from 1 to 50. • Demonstrate the counting order of numbers up to 50. • Count up to 50 <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> • Interact with a two-way frequency table. • Engage with a group of students to gather real-world data and observe the data being organized into a two-way table. • Interact with situations involving greater than, equal to, or less than.


				Learning Progression <i>Building the Base & Engagement</i>
<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
Most Complex ←————→ Least Complex				
<p>S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</p> <p>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. (A2, M3)</p> <p>b. Informally assess the fit of a function by discussing residuals. (A2, M3)</p> <p>c. Fit a linear function for scatterplot that suggests a linear association. (A1, M1)</p>	<p>S.ID.6a Create a scatter plot to represent given or collected data and interpret the relation between the two variables as positive, negative, or no correlation.</p>	<p>S.ID.6b Create a scatter plot for a given data set. (Limited to 8 data points.)</p>	<p>S.ID.6c Interpret the relation between two variables in a scatter plot as positive, negative, or no correlation.</p>	<ul style="list-style-type: none"> S.ID.1b: Create a dot plot to represent given or collected data. Correctly select one answer choice from three options, with the correct answer not always being in the last position. Recognize an ordered pair (x, y) as a point on the coordinate plane. Recognize a point on a coordinate plane. Identify the x- and y- axes in the coordinate plane. Recognize that on a number line the spaces need to be counted not the grid lines, assuming a scale of 1 Recognize that the x- and y- axes are number lines <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with two categorical quantitative data within context (e.g., shoe size /age, age/number of hot dogs eaten) Interact with a scatter plot. Engage with a group of students to gather real-world data and observe the data being organized into a scatter plot. Interact with no more than 3 answer choices be able to select 1 from different positions.
<i>Interpret linear models.</i>				

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Most Complex ←————→ Least Complex				
S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	S.ID.7a Interpret in a real-world context a line of best fit with a given slope and y-intercept for a scatter plot.	S.ID.7b Identify the y-intercept and slope of a line of best fit for a scatter plot.	S.ID.7c Match a line graph with a given data set.	<p>[Between level c and b:</p> <ul style="list-style-type: none"> Create a graph to a story.] Tell stories of a given graph in context. Experience the creation of graphs using science probes. Recognize a line on a coordinate plane. Understand the labels on a coordinate plane. Recognize that the x- and y- axes are number lines. Identify the x- and y- axis. Recognize patterns of the line going up or down, or staying at the same level. Read the graph from left to right. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Interact with two categorical quantitative data within context (e.g., purchasing items and relating an increasing cost, or relating a decreasing amount of money left in the wallet/purse with number of items bought). Interact with a line graph. Engage with a group of students to gather real-world data and observe the data being organized into a line graph.


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Most Complex ←————→ Least Complex				
				<ul style="list-style-type: none"> Interact with no more than 3 answer choices be able to select 1 from different positions.
S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.	S.ID.8a Construct data plots to identify strong and weak correlations of given data.	S.ID.8b Identify the strongest and weakest correlations given visual representation of data.	S.ID.8c Identify the strongest correlation given a visual representation of data.	Not on BP
S.ID.9 Distinguish between correlation and causation.	S.ID.9a Describe real-world situations that illustrate correlation and/or causation (e.g., rain = umbrella).	S.ID.9b Name correlation in real-world example.	S.ID.9c Identify correlation and causation in real-world examples (e.g., shoe size vs. height).	Not on BP
<i>Making Inferences and Justifying Conclusions</i>				
<i>Understand and evaluate random processes underlying statistical experiments.</i>				
S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	S.IC.1a Determine if the given data could come from a specific data-generating device (spinner, coin, number cube).	S.IC.1b Determine the likelihood (likely, impossible, unlikely, and certain) of outcomes from a data-generating device.	S.IC.c1 Determine the likelihood (certain or impossible) of an outcome from a data-generating device.	Not on BP. This standard is taught in Algebra 2. See http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics/Ohio-s-Learning-Standards-in-Mathematics/Transitioning-to-the-2017-Learning-Standards-in-Ma
S.IC.2 Decide if a specified model is consistent with results from a given data-generating process,	S.IC.2a Understand a probability of 0 as impossible, a	S.IC.2b Understand a probability near 0 as	S.IC.2c Understand a probability near 0 as unlikely and	Not on BP. This standard is taught in Algebra 2.

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e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?	probability of 1 as certain, a probability near 0 as unlikely, near 1 as likely, and near 1/2 as equally likely.	unlikely and near 1 as likely.	near 1 as likely using a number line.	
<i>Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</i>				
S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	S.IC.3a Identify sample surveys, experiments, and observational studies.	S.IC.3b Identify a sample survey and an experiment.	S.IC.3c Identify a sample survey.	Not on BP. This standard is taught in Algebra 2.
S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	S.IC.4a Estimate the mean of data given in a sample survey.	S.IC.4b Determine the mean of data given in a sample survey.	S.IC.4c Match the mean of data given in a sample survey.	Not on BP. This standard is taught in Algebra 2.
S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between sample statistics are statistically significant.	S.IC.5a Compare data of a randomized experiments to determine outcome differences.	S.IC.5b Determine if a given treatment changed the outcome of an experiment.	S.IC.5c Match the given treatment that changed the outcome (e.g., bleach changed the stain, water did not).	Not on BP. This standard is taught in Algebra 2.
S.IC.6 Evaluate reports based on data.	S.IC.6a Evaluate if data supports the claim/results.	S.IC.6b Determine if data supports the results/claim.	S.IC.6c Match the data to the given results.	Not on BP. This standard is taught in Algebra 2.

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<i>Learning Standard</i>	<i>Complexity a</i>	<i>Complexity b</i>	<i>Complexity c</i>	
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	Evaluate given data to determine results.			
Conditional Probability and The Rules of Probability				
<i>Understand independence and conditional probability, and use them to interpret data.</i>				
S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).	S.CP.1a Calculate the probability of given event.	S.CP.1b List all possible outcomes of an event.	S.CP.1c Choose the possible outcomes of an event (e.g., 4 possible colors spun on a 4-section spinner).	<ul style="list-style-type: none"> Play games involving probability with or without technology. In a given context identify the number of likely outcomes to be able to choose a fitting simulation, e.g., Will it be a boy or a girl? - I can use a penny (heads/tails) to simulate the situation. There are four colors of a candy in a bag, what is the probability of picking a red candy? Know what an outcome is. Know the language of “likely” and “not likely”. <p>Engagement Statements (demonstration of engaged in the topic)</p> <ul style="list-style-type: none"> Observe and engage with demonstrations of equal probability situations. Engage with a group of students and interact with e.g., spinners and number cubes, to experience outcomes. Interact with 2, 4 or 6 outcomes with equal probability from real-world situations.

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S.CP.2 Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	S.CP.2a Create a Venn Diagram given categorical data.	S.CP.2b Calculate probabilities based on a Venn Diagram.	S.CP.2c Arrange given data into a Venn Diagram (e.g., sports teams).	Not on BP
S.CP.3 Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	S.CP.3a Calculate conditional probabilities of events from Venn Diagrams using the addition rule (e.g., the probability of students who like horror or comedy movies who also like pizza).	S.CP.3b Calculate conditional probabilities of events. (e.g., chance of drawing an ace or face card).	S.CP.3c Arrange given data into a Venn Diagram.	Not on BP
S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. <i>For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly</i>	S.CP.4a Create a two-way frequency table when given data and calculate the probability of an event.	S.CP.4b Given a two-way frequency table, calculate the probability of an event.	S.CP.4c Complete a two-way frequency table given data.	Should be worked on in conjunction with S.ID.5. <ul style="list-style-type: none"> • Enter the data numerically in the two - way table. • Create a table with labels for columns and rows. • Sort cards with data into 4 cells of the table • Gather the data on e.g. index cards. • Recognize a question that involves two categories (e.g. gender/music style, gender/pet ownership.)

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<i>selected student from your school will favor science given that the student is in 10th grade. Do the same for other subjects and compare the results.</i>				Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Interact with a two-way frequency table. Engage with a group of students to gather real-world data and observe the data being organized into a two-way table. Interact with situations involving greater than, equal to, or less than.
S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i>	S.CP.5a Given a real-world scenario, student will name the conditional probabilities and their effects.	S.CP.5b Given a real-world scenario, student will name the conditional probabilities.	S.CP.5c Given a real-world scenario, student will determine if the situation or event is conditional or independent.	Should be worked on in conjunction with S.CP.1. <ul style="list-style-type: none"> Experience and engage in a context where the two events have conditional probability as well as in a context where the two events are independent, e.g., What is the chance of receiving a reward if you follow the rules? (dependent) and What is the chance of not getting homework if you compliment the teacher's attire? (independent) In a familiar context identify the number of possible outcomes. Engagement Statements (demonstration of engaged in the topic) <ul style="list-style-type: none"> Observe and engage with demonstrations of equal probability situations.

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Most Complex  Least Complex				
				<ul style="list-style-type: none"> Engage with a group of students and interact with e.g., spinners and number cubes, to experience outcomes.
<i>Use the rules of probability to compute probabilities of compound events in a uniform probability model.</i>				
S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.	S.CP.6a Given a Venn Diagram or a table with "a given b" statement, identify "a" and "b".	S.CP.6b Given a Venn Diagram or a table, distinguish dependent and independent events.	S.CP.6c Given a Venn Diagram or a table and data, correctly input missing data on the table.	Not on BP
S.CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.	S.CP.7a Write the Addition Rule equation given a complete two-way table.	S.CP.7b Name/identify 2 missing condition variables and input into equation.	S.CP.7c Given a two-way table and the Addition Rule with missing condition, student will identify one missing variable.	Not on BP

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